Mathematical Modelling and New Theories of Learning

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Introduction

Mathematics educators have proposed that students receive opportunities to use and apply mathematics and to engage in mathematical modelling (Blum & Niss, 1991; Schoenfeld, 1985; 1992). Such proposals have emanated, in part, from the positive experiences educators have had when working with students who were engaged in modelling, and the increased opportunities for understanding that such situations appeared to provide. In a parallel shift, psychologists and others concerned with learning have claimed that students need to engage in situations in which they can develop meaning from the applied use of content knowledge. Constructivists claim that students do not simply learn by being told, and that all students should receive opportunities to construct and recontextualise knowledge from meaningful learning experiences (Lerman, 1996; Brandsford, Brown, & Cocking, 1999). Constructivist theories differed from the behaviourist theories offered at the turn of the twentieth century, but like their predecessors they represented knowledge as something that is constructed within people's heads. More recently situated theories of learning (Lave, 1988; Greeno & MMAP, 1998) have offered a new perspective on the development and use of knowledge that pertains in interesting ways to the provision of opportunities for mathematical modelling. Situated theories have taken the focus off individuals, suggesting that knowledge emerges as a series of interactions between people and the world. This suggests that considerations of competency need to examine the ways in which students engage in different *practices*. Thus, it becomes important to engage students in opportunities to use and apply knowledge, not only because such opportunities may afford the development of deeper knowledge, but because students engage in *practices* that they will need to use elsewhere. In this paper I will consider the implications of this perspective on learning for mathematical modelling and problem solving, by casting a situated lens on data collected in a longitudinal, three-year study of 300 students who learned mathematics in very different ways.

Research Methods and Sites.

In a detailed, longitudinal study, in England (Boaler, 1997, 1998, 1999), I monitored approximately 300 students as they attended two schools that employed different teaching approaches. The students were matched at the beginning of the study by ethnicity, gender, social class and prior attainment. The students had followed the same mathematics teaching approaches when they were 11 and 12 years old, then at 13 their mathematical pathways diverged significantly, with one group of students attending a school that used traditional methods, the other group attending a school in which mathematics was taught through problem solving and mathematical modelling. During the 3-year study I observed over 100, one-hour lessons in each school. I interviewed teachers and students and collected students' views through questionnaires that I administered each year. I also gave the students a range of different assessments and analysed their responses to the national mathematics examination that almost all students take at age 16 (GCSE).

At the more traditional school that I have called Amber Hill, the students were taught mathematics using textbooks that asked a series of short, closed questions. Lessons began with methods and techniques being demonstrated by teachers from the front of the room, students would then practice the methods as they worked through their books. The school was disciplined and well organized, students worked hard in lessons and they completed a lot of work. Students were organized into eight 'ability' groups at the school, from set 1 (the highest) to set 8 (the lowest).

At the school I called Phoenix Park, lessons were organised very differently. The mathematics department taught using a series of open-ended projects that they had designed themselves. Students were taught in mixed ability groups and lessons were much more relaxed (for further information, see Boaler 1997a). Some examples of the Phoenix Park projects are listed below:

- ♦ Find the maximum area of a pen made from 36 fences.
- ♦ Play the game of *Yahtzee!* Work out probabilities and consider the use of different strategies.
- ♦ Map the locus of points drawn onto different shapes that are 'rolled' along the floor.
- ♦ Find shapes with an area of 36 and figures with a volume of 216.

The Phoenix Park approach was based on the philosophy that students should encounter situations in which they needed to *use* and *apply* mathematical methods. If the students encountered a need to know a method that they had not met before, the teachers taught it to them within the context of their projects. In the project on 36-fences for example, some of the students found that the biggest area is provided by a 36-sided shape. They needed to learn about trigonometric ratios to find the area of the shape, and so the teacher taught them about trigonometry in order to solve the problem. The Phoenix Park teachers chose the projects to be open, partly to give the students opportunity to choose their own methods and directions, and partly to enable students of different backgrounds and attainment levels to work on the problems.

The students at the two schools therefore received very different opportunities to learn mathematics. At Amber Hill students learned to repeat methods in a standard format, and to interpret a range of classroom cues that helped them know which method to use. The students worked very hard and completed a large amount of work. At Phoenix Park the students learned to choose and adapt different methods, and to hold mathematical discussions. They learned in a more relaxed way. I studied the impact of these different approaches for three years, as the students moved from age 13 to age 16. In the next section I will report the main results of the study.

<u>Research Results.</u>

One of the findings of this three-year study was that students' knowledge development in the two schools was *constituted* by the pedagogical practices in which they engaged. Thus the different practices such as working through textbook exercises, in one school, or discussing and using mathematical ideas, in the other, were not merely vehicles for the development of more or less knowledge, they shaped the forms of knowledge produced. The students at Amber Hill who had learned mathematics working through textbook exercises, performed well in similar textbook situations, but found it difficult using mathematics in open, applied or discussion based situations. The students at Phoenix Park who had learned mathematics through open, group-based projects developed more flexible forms of knowledge that were useful in a range of different situations, including conceptual examination questions and authentic assessments. The students at Phoenix Park significantly outperformed the students at Amber Hill on the national examination, despite the fact that their mathematical attainment had been similar three years earlier, before the students at Phoenix Park embarked upon their open-ended approach. In addition, the national examination was unlike anything to which the Phoenix Park students were accustomed.

One of the indications of the differences in the students' knowledge at the two schools was shown by an analysis of their performance on the national examination. I had divided all the questions on the examination into two categories – conceptual and procedural (Hiebert, 1986), and then recorded the marks each student gained for each question. At Amber Hill the students gained significantly more marks on the procedural questions (which comprised two-thirds of the examination papers) than the conceptual questions. At Phoenix Park, there were no significant differences in the students' performance on the conceptual and procedural questions, even though the conceptual questions were, by their nature, often more difficult than the procedural questions. The Phoenix Park students also solved significantly more of the conceptual questions than the Amber Hill students.

The students at the two schools also developed very different beliefs about mathematics. I interviewed forty students from each school and talked with them about their mathematical beliefs, asking them whether they used mathematics in their day-to-day lives. All the students at both schools said that they did, some of them had part-time jobs outside of school that they described. When I asked the students whether the mathematics they used outside school was similar or different to that which they used inside school, the students at the two schools gave very different responses. All of the Amber Hill students said that it was completely different, and that they would never make use of any of the methods they used in school:

> JB: When you use maths outside of school, does it feel like when you do maths in school or does it feel....
> K: No, it's different.
> S: No way, it's <u>totally</u> different. (Keith and Simon, Amber Hill, year 10, set 7)

> R: Well when I'm out of school the maths from here is nothing to do with it to tell you the truth.

JB: What do you mean?
R: Well, it's nothing to do with this place, most of the things we've learned in school we would never use anywhere. (Richard, Amber Hill, year 10, set 2)

The students at Amber Hill seemed to have constructed boundaries around their knowledge (Siskin, 1994) and they believed that school mathematics was useful in only one place – the classroom. The students at Phoenix Park responded very differently and three-quarters of the students said that there were no differences between mathematics of school and the real world, and that in their jobs and lives they thought back to their school mathematics and made use of it:

JB: When you do something with maths in it outside of school does it feel like when you are doing maths in school or does it feel different?
G: No, I think I can connect back to what I done in class so I know what I'm doing.
JB: What do you think?
J: It just comes naturally, once you've learned it you don't forget. (Gavin and John, Phoenix Park, year 10, MC)

H: In books we only understand it as in the way how, what it's been set, like this is a fraction, so alright then.
L: But like Pope's theory I'll always remember – when we had to draw something, I'll always remember the projects we had to do.
H: Yeah they were helpful for things you would use later, the projects. (Linda and Helen, PP, year 10, MC)

D: I think back to here.
JB: Why do you think that?
A: I dunno, I just remember a lot of stuff from here, it's not because it wasn't long ago, it's just because .. it's just in my mind. (Danny & Alex, Phoenix Park, year 11, JC)

The students gave descriptions of the different and flexible ways they used mathematics that were supported by their positive performance in a range of different assessments (Boaler, 1998).

One conclusion that may be drawn from that study, that would fit with cognitive interpretations of learning, would be that the students in the traditional school did not learn as much as the students who learned mathematics through open-ended projects, and they did not understand in as much depth, thus they did not perform as well in different situations. That interpretation is partly correct, but it lacks important subtleties in its representation of learning. A different analytical frame, that I found useful, was to recognize that the students learned a great deal in their traditional mathematics classrooms at Amber Hill – they learned to watch and faithfully reproduce procedures and they learned to follow different textbook cues that allowed them to be successful as they worked through their books. Problems occurred because such practices were not useful in situations outside the classroom:

> A: It's stupid really 'cause when you're in the lesson, when you're doing work - even when it's hard - you get the odd one or two wrong, but most of them you get right and you think well when I go into the exam I'm gonna get most of them right, 'cause you get all your chapters right. But you don't. (Alan, AH, year 11, set 3)

Problems occurred for the Amber Hill students because their classroom practices were highly specific to the mathematics classroom and when they were in different situations, they became confused, because they tried to follow the cues they had learned in the classroom and discovered that this practice did not help them:

> G: It's different, and like the way it's there like - not the same. It doesn't like tell you it, the story, the question; it's not the same as in the books, the way the teacher works it out. (Gary, Year 11, Set 4).

In other situations, such as their out-of-school jobs and everyday lives, the Amber Hill students engaged in communities that were sufficiently different for them to regard the mathematics they had learned within school as an irrelevance. At Phoenix Park the students were able to use mathematics in different situations because they understood the mathematical methods they met, but also because the practices in which they engaged in the mathematics classroom were present in different situations (Boaler,1999). In class they adapted and applied mathematical methods, and they discussed ideas and solutions with different people. When they used mathematics in the 'real world' they needed to engage in similar practices and they readily did so, drawing upon the mathematics they learned in school. At Amber Hill the students engaged in an esoteric set of practices that were not represented elsewhere and that reduced their ability and propensity to use the mathematics they had 'learned' in different situations.

Discussion and Conclusion.

One of the main conclusions I drew from that three year study was that knowledge and practices are intricately related and that studies of learning need to go beyond knowledge to consider the practices in which students engage and in which they need to engage in the future. There is a pervasive public view that different teaching pedagogies influence the amount of mathematics knowledge students develop. But students do not only learn knowledge in mathematics classrooms, they learn a set of practices and these come to define their knowledge. If students only ever reproduce standard methods that they have been shown, then most of them will only learn that particular practice of procedure repetition, which has limited use outside the mathematics classroom.

Opportunities to engage students in varied practices are not only provided by teaching approaches that have been based on ideas of situativity. Mathematics classrooms that have been designed to provide occasions for mathematical modelling engage students in similar practices, and have done for years (Campbell, 1996; Stocks & Schofield, 1994). But according to a situated perspective, such experiences not only enhance individual understanding; they also provide students with opportunities to engage in practices that are represented and required in everyday life. Educators have proposed that students engage in mathematical modelling as such learning situations can encourage students to develop a deeper, conceptual knowledge of mathematics. I have proposed in this paper that such opportunities also provide students with an opportunity to engage in important mathematical practices that have value beyond the mathematics classroom.

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