The development of *disciplinary relationships*: knowledge, practice, and identity in mathematics classrooms,

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Introduction.

Over the last ten years I have studied the learning opportunities provided to students in different mathematics classrooms, with different teaching approaches. The goal of these studies has been to understand the ways in which the different approaches have shaped students' knowledge of mathematics, and to begin to tease apart the complex relationships between teaching and learning, between knowledge and practice, and between learning and believing. This has provided me with the opportunity to learn about learning, as I have been fortunate enough to watch thousands of mathematics lessons, and analyze students' mathematical development as it has progressed over time. I have done this at the time of what some have described as a cognitive revolution (Schoenfeld, 1999; Resnick, 1993) as views of learning have radically shifted and changed. In this paper I will set out some of the changed perspectives on learning and 'knowledge transfer' that I have developed through my studies in England and California, describing a little of three different studies. I will document a path through my own learning about learning in order to trace an expansion of the dimensions that I have come to believe constitute the learning experience.

For many years educational theories have been based upon the assumption that knowledge is a relatively stable, individual characteristic that people develop and carry with them, transferring from place to place. Knowledge, in such theories, 'consists of coherent islands whose boundaries and internal structures exist, putatively, independently of individuals' (Lave, 1988, p43). Behaviorists, for example, proposed that the best way for people to learn mathematics would be to gain multiple opportunities to practice methods, thus re-enforcing certain behaviors (Greeno & MMAP, 1998). This view was based on an assumption that students learned what was taught, and that knowledge that was clearly communicated and received would be available for use in different situations. Constructivists offered a very different perspective, opposing the view that learners simply receive what is taught, proposing instead that students need to make sense of different ideas and actively organize them into their own cognitive schema, selecting, adapting and reorganizing knowledge as part of their own constructions (Lerman, 1996). Both of these perspectives on learning, as different as they are, represent knowledge as a characteristic of people that may be developed and then used in different situations.

Situated perspectives on learning offer a radically different interpretation, representing knowledge, not as an individual attribute, but as something that is distributed between people and activities and systems of their environment (Lave, 1988; Greeno & MMAP, 1998; Boaler, 2000; Cobb, 2000). This perspective emerged from recognition that people use knowledge differently in different situations and that knowledge, rather than being a stable, individual entity, is co-constructed by individuals and by other people with whom they are interacting and aspects of the situation in which they are working. One of the implications of this shift in the representation of knowledge has been a focus upon the practices and activities of learning. Cognitive structures are still considered (Greeno, 1997), but these are not abstracted out of their learning environments, they are examined as part of the broader system in which they emerge (Greeno & MMAP, 1998). The idea that knowledge is not the sole property of individuals has been viewed suspiciously by some scholars as we have a long history of viewing knowledge differently. But in this paper I would like to propose that these recent views of knowledge have profound practical implications for students' learning of mathematics, some of which I will explore by reviewing three different studies, considering the different dimensions of learning that each study served to highlight.

The Relationships between Knowledge and Practice.

In England I conducted a three-year study of students learning mathematics in two schools (Boaler, 1997). The schools catered to similar populations of students, in terms of ethnicity, gender, social class and prior mathematical attainment, but they taught mathematics in totally different ways. One of the schools – Amber Hill – used a traditional approach to the teaching of mathematics, based upon teacher demonstration and student practice. The other school – Phoenix Park – required students to work on 2-3 week long, open-ended projects that the teachers had designed. The aim of my research was to conduct a detailed investigation into the relationship between teaching approach, student beliefs and student understanding in the two schools. I therefore monitored a cohort of students (approximately 300 in all) over a three-year period, from when they were 13 to when they were 16. A variety of qualitative and quantitative methods were employed, including approximately 100 one-hour lesson observations in each school; questionnaires given to 300 students each year; in-depth interviews with 4 teachers and 40 students from each school; and a range of open, closed and authentic assessments. I also conducted analyses of the students' responses to the national school leaving examination in mathematics. There were no significant differences in the mathematical attainment of the two cohorts of students when the study began.

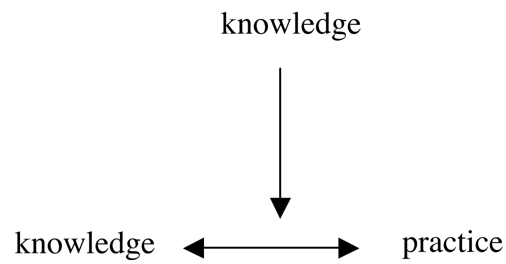
One of the findings of that three-year study was that students' knowledge development in the two schools was *constituted* by the pedagogical practices in which they engaged. Thus it was shown that practices such as working through textbook exercises, in one school, or discussing and using mathematical ideas, in the other, were not merely vehicles for the development of more or less knowledge, they shaped the forms of knowledge produced. One outcome, was that the students at Amber Hill who had learned mathematics working through textbook exercises, performed well in similar textbook situations, but found it difficult using mathematics in open, applied or discussion based situations. The students at Phoenix Park who had learned mathematics through open, group-based projects developed more flexible forms of knowledge that were useful in a range of different situations, including conceptual examination questions and authentic assessments. The students at Phoenix Park significantly outperformed the students at Amber Hill on the national examination, despite the fact that their mathematical attainment had been similar three years earlier, before the students at Phoenix Park embarked upon their open-ended approach (Boaler, 1997). In addition, the national examination was unlike anything to which the Phoenix Park students were accustomed.

One conclusion that may be drawn from that study, that would fit with cognitive interpretations of learning, would be that the students in the traditional school did not learn as much as the students who learned mathematics through open-ended projects, and they did not understand in as much depth, thus they did not perform as well in different situations. That interpretation is partly correct, but it lacks important subtleties in its representation of learning. A different analytical frame, that I found useful, was to recognize that the students learned a great deal in their traditional mathematics classrooms at Amber Hill. They learned to watch and faithfully reproduce procedures and they learned to follow different textbook cues that allowed them to be successful as they worked through their books. Problems occurred because such practices were not useful in situations outside the classroom:

> A: It's stupid really 'cause when you're in the lesson, when you're doing work - even when it's hard - you get the odd one or two wrong, but most of them you get right and you think well when I go into the exam I'm gonna get most of them right, 'cause you get all your chapters right. But you don't. (Alan, AH, year 11, set 3)

One of the main conclusions I drew from that study was that knowledge and practices are intricately related and that studies of learning need to go beyond knowledge to consider the practices in which students engage and in which they need to engage in the future (Boaler, 1999). There is a pervasive public view that different teaching pedagogies only influence the amount of mathematics knowledge students develop. If this were true, then it may make sense to teach all mathematics through demonstration and practice, as the Amber Hill teachers did, as that is probably the most 'efficient' way to impart knowledge. But students do not only learn knowledge in mathematics classrooms, they learn a set of practices and these come to define their knowledge (Dowling, 1996). If they only ever reproduce standard methods that they have been shown, then most students will only learn that particular practice of procedure repetition, which has limited use outside the mathematics classroom. Thus, I concluded from that study that students at Phoenix Park were able to use mathematics in different situations partly because they understood the mathematical methods they met, but also because the practices in which they engaged in the mathematics classroom were present in different situations (Boaler, 1999). I therefore moved from thinking about mathematical capability as a function only of knowledge to viewing it as a complex relationship between knowledge and practice. Figure 1 represents that shift:

Figure 1.



The situated lens that I employed in that study opened two important avenues of exploration and understanding. First, it suggested a focus on classroom practices, pushing me to consider the relationship between students' knowledge production and the characteristics of their teaching and learning environments. Second, it helped me to understand that students did not learn *less* at Amber Hill, they learned *different* mathematics and that my understanding of the students' mathematical learning opportunities and capabilities at the two schools, needed to extend beyond knowledge to the practices in which students engaged in the classroom *and the relationship between the two.* The site of knowledge transfer had shifted, in my understanding, from students' minds to the mathematical practices in which they engaged. Thus the Phoenix Park students were more able to 'transfer' mathematics, not because their knowledge was secure and available for transport, but because they engaged in a set of practices in the classroom that were present elsewhere.

This more complex representation of learning as a relation between knowledge and practice seemed generative, but future studies of mathematics teaching and learning in which I engaged revealed a need to further expand my conceptions of learning to include a third dimension, that goes beyond knowledge and practice.

Relationships between Knowledge, Practice and Identity.

In a recent study in which I and fellow researchers interviewed eight students from each of 6 Northern Californian high schools, I was given further opportunities to investigate the nature of learning in different teaching environments (Boaler & Greeno, 2000). The 48 students we interviewed were all attending advanced placement (AP) calculus classes. In that study four of the schools taught using traditional pedagogies similar to those at Amber Hill – the teachers demonstrated methods and procedures to students, who were expected to reproduce them in exercises. In the other two schools, students used the same calculus textbooks, but the teachers did not rely on demonstration and practice, they asked the students to discuss the different ideas they met, in groups. In that study we found that students in the more traditional classes were offered a particular form of participation in class that we related to Belencky, Clinchy, Goldberger, & Tarule's notion of 'received knowing' (1986, p4). Mathematics knowledge was *presented* to students and they were required to learn by attending carefully to both teachers' and textbook demonstrations. The mathematical authority in the classrooms was external to the students, resting with the teacher and the textbooks (Ball, 1993), and the students' knowledge was dependent upon these authoritative sources. In these classrooms it seemed that the students were required to receive and absorb knowledge from the teacher and textbook and they responded to this experience by positioning themselves as *received knowers* (Belencky et al, 1986).

The students who were learning in these traditional classrooms were generally successful, but we found that many students experienced an important conflict between the practices in which they engaged, and their developing identities as people. Thus many of the students talked about their dislike of mathematics, and their plans to leave the subject as soon as they were able, not because of the cognitive demand, but because they did not want to be positioned as received knowers, engaging in practices that left no room for their own interpretation or agency. The students all talked about the kinds of person (Schwab, 1969) they wanted to be – people who used their own ideas, engaged in social interaction, and exercised their own freedom and thought, but they experienced a conflict between the identities that were taking form in the ebb and flow of their lives and the requirements of their AP calculus classrooms:

> K: I'm just not interested in, just, you give me a formula, I'm supposed to memorize the answer, apply it and that's it.
> Int: Does math have to be like that?
> B: I've just kind of learned it that way. I don't know if there's any other way.
> K: At the point I am right now, that's all I know. (Kristina & Betsy, Apple school)

The disaffected students we interviewed were being turned away from mathematics because of pedagogical practices that are unrelated to the nature of mathematics (Burton, 1999a, b). Most of the students who told us about their rejection of mathematics in the 4 didactic classrooms – 9 girls and 5 boys, all successful mathematics students – had decided to leave the discipline because they wanted to pursue subjects that offered opportunities for expression, interpretation and human agency. In contrast, those students who remained motivated and interested in the traditional classes were those who seemed happy to 'receive' knowledge and to be relinquished of the requirement to think deeply:

> J: I always like subjects where there is a definite right or wrong answer. That's why I'm not a very inclined or good English student. Because I don't really think about how or why something is the way it is. I just like math because it is or it isn't. (Jerry, Lemon school)

The students in didactic classes who liked mathematics did so because there were only right and wrong answers, and because they did not have to consider different ideas and methods. They did not need to think about 'how or why' mathematics worked and they seemed to appreciate the passive positions that they adopted in relation to the discipline. For the rest of the students in the traditional classes, such passive participation was not appealing and this interfered with their affiliation and their learning.

In the other two calculus classes in which teachers engaged students in mathematical discussions, a completely different picture emerged. In the discussion oriented classes the students had formed very different relationships with mathematics that did not conflict with the identities they were forming in the rest of their lives. The students in these classes regarded their role to be learning and understanding mathematical relationships, they did not perceive mathematics classes to be a ritual of procedure reproduction. This lack of conflict was important – it meant that the students who wanted to do more than receive knowledge, were able to form plans for themselves as continued mathematics learners. The student quoted below is just one of those we interviewed in the discussion-oriented calculus classes that planned to major in mathematics:

> Sometimes you sit there and go 'it's fun!' I'm a very verbal person and I'll just ask a question and even if I sound like a total idiot and it's a stupid question I'm just not seeing it, but usually for me it clicks pretty easily and then I can go on and work on it. But at first sometimes you just sit there and ask – 'what is she teaching us?' 'what am I learning?' but then it clicks, there's this certain point when it just connects and you see the connection and you get it. (Veena, Orange school)

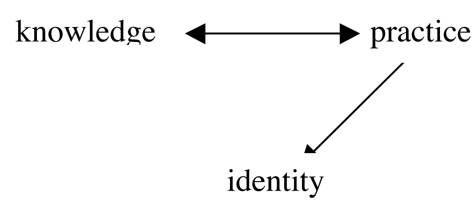
One of the interesting aspects of Veena's statement about mathematics class is her description of herself as a 'verbal person'. This was the reason that many of the students in the more traditional classes gave for rejecting mathematics. Indeed it seems worrying, but likely, that Veena may have rejected mathematics if she had been working in one of the four other schools in which the discussions and connections she valued were under-represented.

The type of participation that is required of students who study in discussion-oriented mathematics classrooms is very different from that required of students who learn through the reception and reproduction of standard methods. Students are asked to contribute to the judgment of validity, and to generate questions and ideas. The students we interviewed who worked in discussion-based environments were not only required to contribute different aspects of their selves, they were required to contribute *more* of their selves. In this small study we found the notion of identity to be important. Students in the different schools were achieving at similar levels on tests but they were developing very different relationships with the knowledge they encountered. Those students who were only required to receive knowledge described their relationships with mathematics in passive terms and for many this made the discipline unattractive. Those who were required to contribute ideas and methods in class described their participation in active terms that were not inconsistent with the identities they were developing in the rest of their lives. Wenger's (1998) depiction of learning as a process of 'becoming' was consistent with the students' reported perceptions:

> 'Because learning transforms who we are and what we can do, it is an experience of identity. It is not just an accumulation of skills and information, but a process of becoming – to become a certain person or, conversely, to avoid becoming a certain person. Even the learning that we do entirely by ourselves contributes to making us into a specific kind of person. We accumulate skills and information, not in the abstract as ends in themselves, but in the service of an identity'. (Wenger, 1998, p. 215)

This was a small study but it served to illuminate the importance of students' relationship with the discipline of mathematics that emerged through the pedagogical practices in which they engaged. This helped my understanding of learning to expand further to include the identities students were developing as learners and as people, as they engaged in different practices (see figure 2).

Figure 2.



But these were early ideas and this representation seemed incomplete, as whilst identity seemed important, and clearly connected with pedagogical practices, I was unclear about the way this notion related to knowledge – the missing side of the triangle in figure 2. A more recent study, as well as the writing of Andrew Pickering (1995), has provided an important site for the continued exploration of these ideas, in particular for the investigation of relationships between knowledge and identity.

<u>Developing Relationships with the Discipline of Mathematics.</u>

The final study that I will describe, in which we are monitoring the learning of approximately 1000 students as they go through three different high schools, is a follow up to that which I conducted in England. Two of the schools offer a choice of mathematics curriculum, which they describe as 'traditional' and 'reform' oriented. In the classrooms that are using a 'reform' approach we observe very different patterns of interaction than those in the more traditional classrooms. As we work to understand the capabilities that are being encouraged by these examples of classroom interaction we are again finding the notion of agency to be important.

The students in the reform classrooms we are studying, as in the Phoenix Park classrooms, are given the opportunity to use and apply mathematics, a process which confers upon them considerable amounts of human agency. Students are required to propose 'theories', critique each other's ideas, suggest the direction of mathematical problem solving, ask questions, and 'author' some of the mathematical methods and directions in the classroom. One conclusion we could draw from these interactions would be that the students have *more* agency than those in more traditional classrooms, but whilst this may be true, such an observation feeds into debates about 'traditional' and 'reform' teaching methods in unfortunate ways. There is a common perception that students in 'reform' curriculum programs simply have more agency and more authority (Rosen, 2000), which often leads to fears that students are not learning enough, that they are left to wander in different, unproductive directions, and that they learn only "fuzzy" mathematics (Becker & Jacob, 2000). But we are finding that the *nature* of the agency in which students engage in these classrooms is related to the discipline of mathematics and the practices of mathematicians in important ways. Such insights have emanated from an analytic frame proposed by Andrew Pickering (1995).

Pickering studied the work of professional mathematicians and concluded that their work requires them to engage in a 'dance of agency' (1995, p116). He proposes that there are different types of agency and that conceptual advances require the interchange of human agency and the 'agency of the discipline' (1995, p116). Pickering considers some of the world's important mathematical advances and identifies the times at which mathematicians use their own agency – in creating initial thoughts and ideas, or by taking established ideas and extending them. He also describes the times when they need to surrender to the 'agency of the discipline', when they need to follow standard procedures of mathematical proof, for example, subjecting their ideas to widely agreed methods of verification. Pickering draws attention to an important interplay that takes place between human and disciplinary agency and refers to this as 'the dance of agency' (1995, p116).

Pickering's framework seems important for our analyses of the different practices of teaching and learning we observe. 'Traditional' classrooms are commonly associated with disciplinary agency, as students follow standard procedures of the discipline. 'Reform' classrooms, by contrast, are associated with student agency, with the idea that students use their own ideas and methods. We see something different in our observations of 'reform' classrooms. Rather than a group of students wandering unproductively, inventing methods as they go, we see a collective engaged in the 'dance of agency'. The students spend part of their time using standard methods and procedures and part of the time 'bridging' (Pickering, 1995, p11) between different methods, and modifying standard ideas to fit new situations.

In many of the traditional classrooms I have observed, in this and previous years, students have received few opportunities to engage in the 'dance of agency', and when they need to engage in that 'dance', in new and 'real world' situations, they are ill prepared to do so. When I interviewed a class of students in the fourth year of the reform program at one of the schools, the students all described an interesting relationship with mathematics that contrasted with the students working at similar levels of mathematics in traditional AP calculus classes. As part of the interviews we asked students what they do when they encounter new mathematical problems that they cannot immediately solve. In the extracts below the students give their responses:

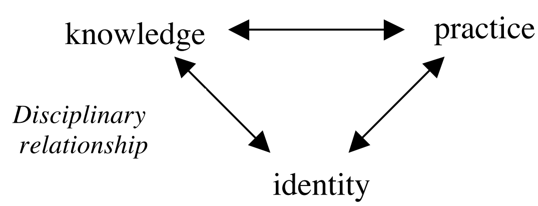
> K: I'd generally just stare at the problem. If I get stuck I just think about it really hard and then just start writing. Usually for everything I just start writing some sort of formula. And if that doesn't work I just adjust it, and keep on adjusting it until it works. And then I figure it out. (Keith)
>
> B: A lot of times we have to use what we've learned, like previous, and apply it to what we're doing right now, just to figure out what's going on It's never just, like, *given*. Like "use this formula to find this answer" You always have to like, change it around somehow a lot of the time. (Benny)

These students seem to be describing a 'dance of agency' as they move between the standard methods and procedures they know and the new situations to which they would apply them. They do not only talk about their own ideas, they talk about adapting and extending methods and the interchange between their own ideas and standard mathematical methods. The student below talks in similar terms as he reflects upon his decision to enter the IMP program:

> E: As far as the thought processes that you use in IMP are different from the standard parroting back of facts about algebra, I just really think that it's changed the way I think about a lot of things besides math that I really appreciate.
> Int: Like what?
> E: Like, if nothing else, it's breaking out of the pattern of just taking something that's given to you and accepting it and just going with it. It's just looking at it and you try and point yourself in a different angle and look at it and reinterpret it. It's like if you have this set of data that you need to look at and find an answer to, you know, if people just go at it one way straightforward you might hit a wall. But there might be a crack somewhere else that you can fit through and get into the meaty part. (Ernie).

Many of the students at Amber Hill frequently 'hit a wall' when they were given mathematics problems to solve. They would try and remember a standard procedure, often using the cues they had learned. If they could remember a method they would try it, but if it did not work, or if they could not see an obvious method to use, they would give up. The students we interviewed in the IMP4 classes described an important practice of their mathematics classroom – that of working at the interplay of their own and disciplinary agencies – that they used in different mathematical situations. Additionally the students seemed to have developed identities as mathematics learners who were willing to engage in the interplay of the two types of agency. The students had developed what we are regarding as a particular *relationship with the discipline of mathematics* that served to complete the triangle in figure 3:

Figure 3.

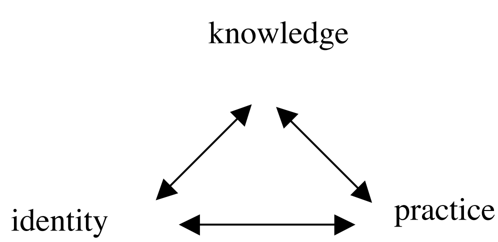


A number of researchers have written about the importance of productive beliefs and dispositions (Schoenfeld, 1992; McLeod, 1992) but the idea of a 'disciplinary relationship' serves to connect knowledge and belief in important ways. Herrenkohl and Wertsch (1999) have suggested a notion that addresses this connection, that they call the 'appropriation' of knowledge. They distinguish between mastery and appropriation, saying that too many analyses have focused only upon students' mastery of knowledge, overlooking the question of whether students 'appropriate' knowledge. They claim that students do not only need to develop the skills they need for critical thinking, they also need to develop a disposition to use these skills. In claiming that students need to 'appropriate' knowledge, they suggest a connection between the content students are learning and the ways they relate to that knowledge. The fact that the Phoenix Park students were able to use mathematics in different situations may reflect the similarity in the practices they met in different places, but it also reflects the fact that they had developed a positive, active relationship with mathematics. They expected to be able to make use of their knowledge in different situations and the identities they had developed as learners included an active relationship with mathematics. This relationship reflected their engagement in the dance of agency. Thus they were able to 'transfer' mathematics, partly because of their knowledge, partly because of the practices in which they engaged, and partly because they had developed an active and productive relationship with mathematics. This idea seems to pertain to theories of learning transfer and expertise in important ways, expanding notions of capability beyond knowledge and practice to the dispositions they produce and the relations between them.

Discussion and Conclusion.

In tracing a path through three of my recent studies I hope to have brought some useful analytic lenses to the question of knowledge transfer and mathematical capability. I have outlined a shift from a focus only upon knowledge, to one that attends to the inter-relationships of knowledge, practice, and identity. This seems to offer new perspectives on knowledge use and capability that fit with the behavior of experts. If we consider a mathematician at work, for example, she may be given a new problem to solve, but lack the knowledge needed to solve it. In such a situation it seems likely that she will still make progress, as she has learned a set of mathematical practices that she may use in trying to solve the problem – practices such as representing the situation graphically, generalizing to different sets of numbers, or 'bridging' from a method she knows. She has also developed a productive relationship with the discipline of mathematics that means she will try different methods, garner helpful resources and make use of the knowledge and practices she has learned. She will work at the base of the triangle in figure 4 in the production of knowledge, yet many analyses of knowledge transfer fail to recognize these dimensions of capability.

Figure 4



The analysis I have offered in this paper leaves many unanswered questions – about the specificity of the relationships that pertain between knowledge and identity for example. Is it enough to develop a productive relationship with the discipline of mathematics or do learners need to appropriate <u>particular</u> knowledge, with relationships pertaining to specific domains of mathematics? Are all learners advantaged by opportunities to engage in a dance of agency, or do some learners advance through a more passive relationship with the discipline? How do identities of race, class and gender intersect with those of mathematics? These and other questions I will continue to consider, but this particular description of my learning trajectory ends with an idea about students' use of mathematics that goes beyond knowledge and practices to the inter-relations of knowledge, practices, and identities that emerge in different environments.

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