



The derailing impact of content standards—an equity focused district held back by narrow mathematics

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ABSTRACT

Many school districts strive to achieve high mathematics achievement and equitable outcomes. This study examines the work of Gateside district, an urban district that had made great progress in the enactment of equity minded policies such as de-tracking and the availability of high-level mathematics to all students. A detailed, case study of teaching and learning in the high school mathematics classrooms showed that the equity focused work of the teachers and district leaders was compromised by the narrow mathematics standards that informed the tasks used in classrooms. The prevalence of narrow tasks led students to develop binary perceptions of each other, revealing a fundamental tension between narrow content standards and equitable outcomes.

1. Introduction

While research in mathematics education continues to highlight the importance of students actively engaged in their learning of mathematics (Boaler, 2002b, 2002a; Boaler & Staples, 2008; Schoenfeld, 2002), the model of mathematics teaching that dominates the US, particularly in the high school years, is one of passive reception of knowledge from a teacher and textbook (Boaler, 2015). This has led to wide-spread underachievement (Boaler, 2006; PISA, 2012), an adult population who fears and avoids mathematics (Ashcraft, 2002; Beilock, 2011; Chestnut, Lei, Leslie & Cimpian, 2018), and insufficient numbers of students available to take on the important STEM needs of society (NCTM, 2014; Wolfram, 2020). In school districts across the US, fixed mathematics pathways – that sort students out of high-level mathematics at an early age – limit students' access to more advanced course-taking sequences and, by proxy, to careers in STEM (Daro & Asturias, 2019). At the classroom level, some teachers also implement pedagogical approaches that reinforce inequitable patterns of participation, learning, and identification with mathematics (Hand, 2010; Louie, 2017). In addition, implicit and explicit messages about mathematics given to students—by parents, the media, and by society more broadly—contribute to the conception of mathematics as an exclusive subject reserved for the “gifted” few (Boaler, 2019; Chestnut et al., 2018).

Tracking, procedural teaching, and fixed messaging have long been established as areas of concern in mathematics education. This paper proposes the role of a fourth, rarely cited source of disengagement and inequity – the nature of the mathematics being taught. Through a detailed study of a large urban district that has made significant and im-

portant change in turning around the three well established factors that limit high achievement, we show the important role played by the content being taught. The district that is the focus of this study has achieved a great deal through changing mathematics pathways, largely eliminating tracking, delivering high quality professional development to teachers, and developing a teaching workforce that is committed to equity. While these achievements are impressive, we found that the work of the teachers and district leaders was undermined by the nature of the mathematics set out in the Common Core State high school standards.

The mathematics in question has been taught for generations, and usually escapes critical observation, but this paper highlights the important role it plays in derailing progress— particularly in the promotion of equitable teaching practices. The Common Core offered a new set of mathematics standards—with notable improvements in the K-8 years. This paper focuses on the teaching of high school algebra, highlighting the ways that the content of high school mathematics limits the possibility of equitable engagement.

2. Background

In most school districts, mathematics is taught through a system of ability grouping or “tracking”— an organizational practice in which mathematics classes in higher tracks offer more advanced content for chosen groups of students (Boaler, 2013; Hand, 2010; Oakes, 1986). In the US this practice typically begins as early as sixth grade, and in many school districts the different classes created lead to different mathematics pathways, through the end of high school, and beyond. Many studies have highlighted the problems with this approach, as students in lower

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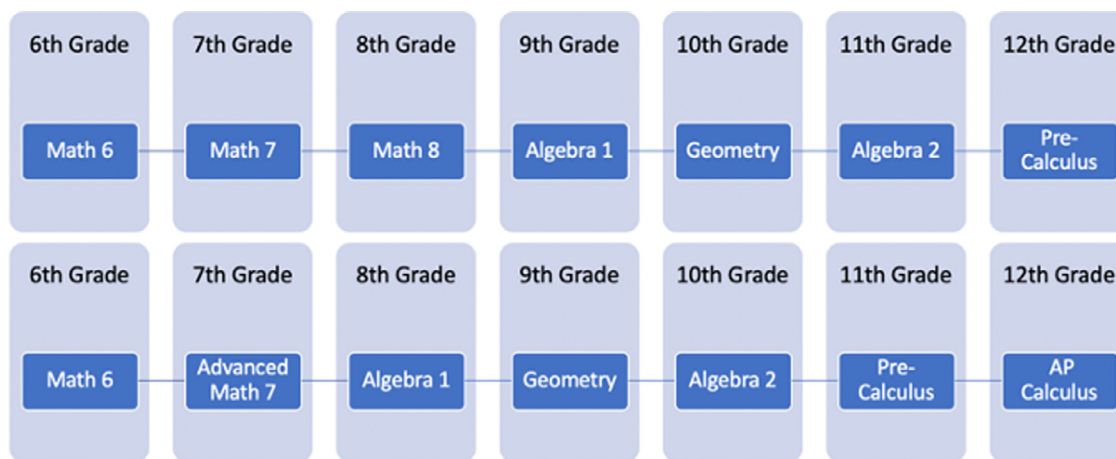


Fig. 1. Typical Mathematics Pathway Options (for students from age 11 to 18).

tracks are taught less rigorous content, often by less qualified teachers, which limits their future opportunities (Boaler, 2006, 2008, 2013; Boaler & Selling, 2017; Boaler & Staples, 2008; Oakes, 1986). Moreover, students of color and students of low-income backgrounds are overrepresented in low level mathematics classes and underrepresented in advanced level mathematics classes, which compounds existing historical inequities (Boaler, 2015; Daro & Asturias, 2019; Hand, 2010).

A 2012 study of nine school districts in California found that large numbers of students were testing proficient in algebra at the end of middle school but being asked to repeat algebra in high school, putting them on a low-level mathematics pathway. An examination of which students were being “held back” showed that they were disproportionately students of color (The Lawyers Committee for Civil Rights, 2013). Other studies have shown that teachers’ subjective beliefs about students can limit Black students’ access to higher mathematics pathways (Campbell, 2012). Black girls in particular face biases due to their race and gender, and when they are denied entry into advanced mathematics courses in middle school, this limits their access to high school and college STEM courses (Joseph, Hailu & Boston, 2017). These inequities not only limit students’ learning opportunities, but actively perpetuate traditional conceptions of mathematics as a subject intended for a minority of elite white or Asian men (Ellis & Berry, 2005).

The practices of tracking and sorting students have long-term consequences beyond schooling (Boaler & Selling, 2017). In many districts, the highest-level mathematics course available is calculus. This pathway has played a large part in college admissions for generations. The “top” universities in the US, that accept less than 25% of applicants, have calculus as an unstated requirement. For example, the Harvard Crimson found that 92% of freshmen had taken AB calculus or higher in high school (Harvard, 2018). But the calculus pathway is characterized by severe inequities. US high schools typically teach four classes as a prelude to calculus (algebra-geometry-algebra 2- pre calculus) which means that students in these schools can only take calculus if they are “advanced” in middle school, taking algebra as an eighth-grade student. Fig. 1 shows two typical mathematics tracks offered by most US school districts, with only one pathway leading to calculus:

Daro and Asturias (2019) point out that students who are not selected for the calculus pathway often “sink into a bog of remediation and ineligibility from which few escape” (p. 7). Not only does the inequitable system that they describe filter students out of mathematics and STEM—particularly Latinx and Black students—it also acts as a gatekeeper to higher education.

Systems that filter students out of mathematics were designed in times when people believed that only certain students have a “math brain,” but it is now well documented that mathematics ability is not defined biologically (Boaler, 2019; Chestnut et al., 2018; Devlin, 2000).

Not only is mathematics often thought of as an exclusive subject, the notions of who is capable of high-level mathematics are too frequently racialized and gendered (Gutiérrez, 2017). Even when schools work to change these ideas, narrow beliefs about mathematics and learning often undermine reform efforts. Louie (2017) building on Parks (2010) looked at four classrooms led by teachers committed to implementing equitable teaching practices through a pedagogical approach called Complex Instruction (CI) (Cohen & Lotan, 1997). The teachers were committed to empowering students whose previous mathematics learning experiences had not served them well. While this group of teachers was consciously working to use inclusive practices and to value a wide variety of mathematical strengths, Louie (2017) found that narrow beliefs about student ability and math learning subconsciously prevailed in the teachers’ practice, reinforcing what she described as a “culture of exclusion.”

Another factor in the inequitable nature of mathematics participation is the teaching approaches often used. When mathematics is taught as a set of procedures to follow, many students disengage, and various studies have shown that procedural teaching is particularly damaging for girls and students of color (Boaler & Sengupta-Irving, 2016; Boaler, Cordero & Dieckmann, 2019). Additionally, procedural teaching encourages students to take a “memorization” approach to mathematics, which has been shown to be associated with low achievement (Boaler & Zoido, 2016; Gray & Tall, 1994; PISA, 2012). Procedural mathematics is also fundamentally different to the creative and multidimensional approach used by mathematicians (Burton, 1998; Strogatz, 2012, 2019; Wolfram, 2020).

In contrast to a procedural teaching and learning approach, teachers who invite students to consider the meaning of mathematical methods, to choose and discuss approaches and to think creatively, enable students to develop a sense of agency and mathematical authority (Amit & Fried, 2005; Boaler, 2002b, 2002c, 2002a; Boaler & Staples, 2008; Cobb, Gresalfi, & Hodge, 2009; Gutstein, 2007). When students are taught that they “belong in mathematics,” and that mathematics is about depth and connections, and they are encouraged to develop growth mindsets, higher and more equitable achievement results (Boaler, 2019; Dweck, 2012; Yeager & Walton, 2011).

3. The setting

In 2014 the district that is the focus of this paper, which we refer to as Gateside Union District, began work to address three established sources of mathematics disengagement and inequality – tracking, procedural teaching, and fixed messaging. In an unusual and commendable initiative, district leaders reviewed research on the nature of the mathematics inequalities they saw in their district and began a series of steps to counter them. This included presenting research to teachers,

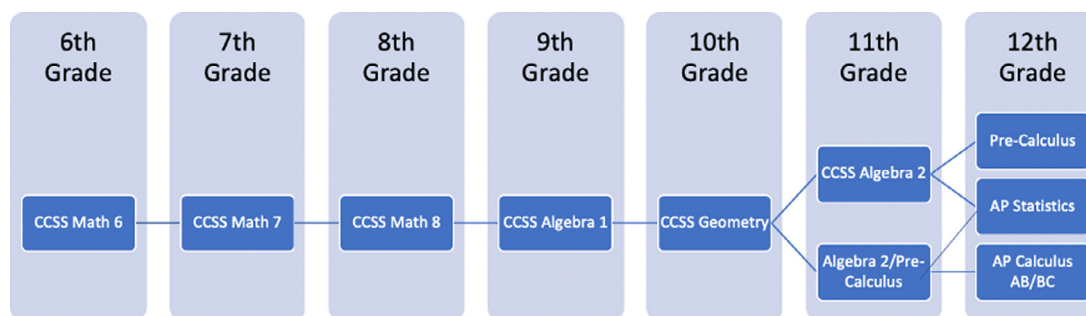


Fig. 2. Gateside District Mathematics Pathway Options.¹

the school board and the local community. Most notably, the district proposed to the school board that they eliminate tracking before 11th grade – a radical plan that was unanimously accepted by the school board. The proposal also included moving algebra to ninth grade for all students and setting out pathways to calculus for all students. The plans were accepted and accompanied by extensive professional development for the high school teachers.

In contrast to the pathway that most districts in the US offer – with middle school mathematics leading to calculus for only some students, Gateside district offered the following mathematics pathways (see Fig. 2). Notably, all students take the same mathematics course until the end of tenth grade.

The creation of a mathematics pathway and set of courses, that brought all students together in the same classes, attracted the opposition that many would expect. This came primarily from parents of currently accelerated students. Groups of parents organized meetings and Facebook groups to oppose the district changes, but the district stood firm and within a year they saw substantial benefits. Under their previously tracked system 40% of students failed algebra, and inequities were very evident – after one year of the new system the algebra failure rate dropped to 8 percent. Algebra I repeat rates dropped for all student subgroups, falling from 52 to 19 percent for African American students and from 57 to 14 percent for Latinx students. These positive results are similar to those of other districts and mathematics departments that have engaged in equity focused work (Burris, Heubert & Levin, 2006; Gutierrez, 1996). One of the arguments of those opposed to equitable initiatives is that equity reduces the chances of high achievers to excel but there was no evidence of this compromise and the proportion of students taking advanced classes at Gateside district increased by one third.¹

Prior to the de-tracking initiative many teachers (including all those teachers included in this study) were given extensive professional development in Complex Instruction (CI), a pedagogical approach that centers upon three principles for creating equity in heterogeneous classrooms through groupwork (Cohen & Lotan, 1997, 2014). The first principle of productive groupwork involves students developing responsibility for each other, serving as academic and linguistic resources for one another (Cabana, Shreve & Woodbury, 2014; Cohen & Lotan, 1997). As part of this development of student authority, students learn to be resources for each other as they serve in different roles as: group facilitator, resource manager, recorder/reporter or team captain.

The second principle of CI is students working together to complete tasks (Cohen & Lotan, 2014). To realize this principle, teachers must manage equal participation in groups by valuing and highlighting a wide range of abilities and attending to issues of status amongst students. Teachers widen the traditional definitions of valuable skills by publicly recognizing the wealth of “intellectual abilities” students hold (Cohen & Lotan, 2014; Tsu, Lotan & Cossey, 2014). During groupwork,

the teacher looks for opportunities to elevate students by highlighting their abilities and contributions to the group, which is referred to as “assigning competence” (Boaler & Staples, 2014; Cohen & Lotan, 2014). This principle recognizes the fact that group interactions often create status differences between students – and when a teacher perceives that a student has become “low status” in a group, they intervene by publicly praising a valuable mathematical contribution they have made.

Underlying these two principles is a third: the implementation of multi-dimensional, ‘groupworthy’ tasks, which are challenging, open-ended, and require a range of intellectual abilities (Banks, 2014; Cohen & Lotan, 1997). These tasks are characterized as different than collaborative seat work—work that students typically do individually but are asked to do together—in that the completion of the task necessitates the participation of all group members (Cohen & Lotan, 2014). For a task to be groupworthy in mathematics, it should emphasize thinking and reasoning (Staples, 2014), valuing processes over products (Velazquez & Louie, 2014); such features are described by the originators of Complex Instruction as vital to productivity (Cohen & Lotan, 2014). It is important to note, without the presence of groupworthy tasks, the need for meaningful collaboration is gone and the other two principles of Complex Instruction are threatened.

The important role of multi-dimensional tasks in a CI mathematics classroom conflicts with traditional conceptions of mathematics learning as the rote practice of procedures. As Tsu et al. (2014) note, the idea that mathematical understanding can be demonstrated through narrow mathematics is “incompatible with building an equitable Complex Instruction mathematics classroom” (p. 136). Cabana, Shreve, and Woodbury (2014) offer five aspects of equitable mathematics instruction in a CI classroom:

- (1) Structuring lessons to support engagement in groupworthy tasks
- (2) Approaching math concepts through multiple representations
- (3) Organizing curriculum around big ideas
- (4) Using justification to push students to articulate their mathematical thinking
- (5) Making students’ thinking public and valued.

(2014, p. 40)

It is notable that the five aspects all offer a conception of mathematics as a broad subject that can be encountered as a set of big ideas that centralize student thinking, and multiple representations (Cabana et al., 2014; Horn, 2014).

Studies of careful implementation of CI in schools have shown that it improves student achievement (Boaler & Staples, 2008; Horn, 2007), students’ relationships with mathematics (Velazquez & Louie, 2014), helps build a sense of community among students (Boaler, 2008), and supports vulnerable students to increase engagement and “take up space” (Hand, 2014; Jilk, 2014).

Within the context of these positive outcomes, our study set out to investigate the factors enabling or constraining high and equitable achievement within heterogeneous classrooms.

¹ Note: At the time of the data collection, the district served approximately 4,750 ninth grade Algebra 1 students across 14 high schools.

Student Participants' Achievement Levels

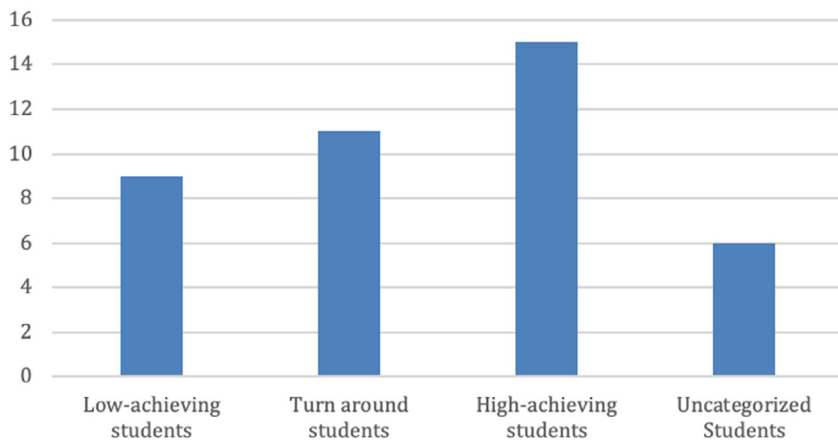


Fig. 3. Demographics of Student Participants, ages 14–15 (n = 41).

4. The study

The goal of our study was to examine the factors that constrain or support equitable participation in heterogeneous classrooms. Our recognition of the social nature of learning led to a situated framework that pays attention to forms of participation and engagement (Lave, 1991, p. 67; Jilk, 2014). Situated theories propose that learning takes place through interaction, participation, and engagement, highlighting the importance of studying the ways students engage in mathematics classrooms. Schools, such as those in our study, exist as culturally situated communities of practice, in which students construct knowledge through interaction with their peers, their teacher, and the mathematics content (Boaler, 2002a; Langer-Osuna, 2015; Lave, 1991; Nasir, Hand, & Taylor, 2008). A focus on social practices enabled us to pay attention to the connection between these heterogeneous mathematics classrooms and the broader district in which they were situated, as we would expect the district’s strong culture around equitable mathematics pathways to shape the learning environments.

In addition to a situated framing we employed an interpretive paradigm that allowed understandings from participants to influence research directions (Taylor & Medina, 2011). An interpretive paradigm centers the voices and experiences of participants - in our case teachers and students. Within this interpretive paradigm, participants’ experiences led to the raising of a factor that we had not expected to be a part of our study (Creswell, 2014). As we came to better understand our participants’ experiences, our data collection evolved to more expansively capture this different factor and the ways that it impacted classroom environments.

4.1. Setting & participants

This study was conducted at two high schools in Gateside District, a large urban school district in California. Gateside District serves a racially diverse student body, with significant Latinx (27%), Asian (35%), and White (15%) populations, as well as smaller (less than 10%) African American Filipinx, Pacific Islander, and American Indian populations. Additionally, approximately 55% of students are considered socioeconomically disadvantaged, 29% of students are designated as Language Learners, and 11% of students have been diagnosed with Special Educational Needs.

To select school sites for this study, the district provided the research team with a recommended list of seven schools (out of a total of 14 high schools in the district) that had fully implemented Complex Instruction and the district’s core curriculum. Each of these school’s principals and math department chairs were contacted to request to observe Algebra 1 classrooms and gauge interest in participating in the study. Two schools

Table 1
Demographics of teacher participants (n = 8).

School	Teacher	Teaching experience (years)
Lewis High School	Anderson	3
	Lang	25
Edison High School	Hirsch	8
	Li	5
	Yang	1
	Zhao	3
	Garcia	15
	Chen	11

Table 2
Criteria for selecting students to interview.

Category	Criteria
Two High Achieving students	<ul style="list-style-type: none"> Students who came to your class high achieving (broadly defined) and have continued to achieve similarly
Two “Turn Around” students	<ul style="list-style-type: none"> Students who you’ve seen experience noticeable growth (broadly defined) Students who are benefitting from a heterogeneous classroom
Two Low Achieving students	<ul style="list-style-type: none"> Students who have achieved at lower levels than most of the class and continue to be low achieving (broadly defined)

were selected based on the numbers of Algebra 1 teachers interested in participating. The research team met with all Algebra 1 teachers at both schools to invite them to join the study; two of the three Algebra 1 teachers at Lewis High School and six of the seven Algebra 1 teachers at Edison High School consented to participate.

The eight participating teachers had a wide range of teaching experience, ranging from one to 25 years, with an average of 8.875 years per teacher (see Table 1). All teachers used Gateway District’s Algebra 1 curriculum and implemented Complex Instruction (CI).

To understand the practices that supported or constrained learning researchers collected and analyzed a range of data, including 16 h of classroom observation, student surveys from 528 algebra 1 students, and interviews with eight teachers and 41 students.

Across the eight classrooms, 41 students were selected for interview, as shown in Fig. 3, based on teacher recommendation. We asked teachers to nominate pairs of students who were high or low achieving or had “turned around” their achievement, as described in Table 2. Students were sampled from across the achievement range to ensure a focus on

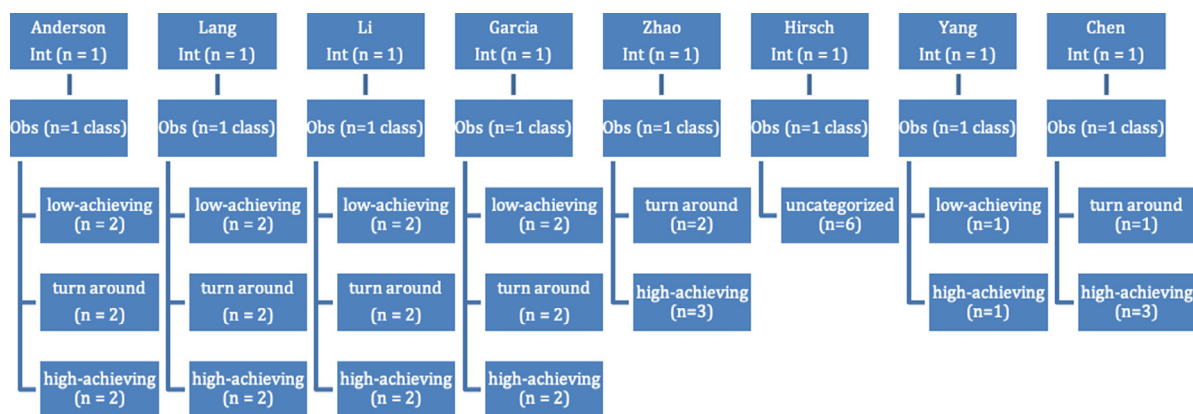


Fig. 4. Data sources (by participants) collected across our two school sites – all students were in 9th grade ages 14–15.

students at different levels of academic achievement. This resulted in interviews with 9 low-achieving students, 11 “turnaround” students, 15 high-achieving students, and 6 students that were uncategorized.

4.2. Data sources

Multiple sources of qualitative data were collected during the 2018–19 school year. Site-based data sources are depicted in Fig. 4 and consist of: a) semi-structured interviews with teachers, which were audio recorded and lasted approximately 45 min, b) observations of one of each teacher’s Algebra 1 classes for two consecutive days, which were video recorded and lasted approximately 1 hour, and c) semi-structured interviews with two to six students from each observed classroom, typically in pairs, which were audio recorded and lasted between 15 and 35 min.

Additionally, districtwide administration of a Mathematical Mindset Survey yielded 528 (out of approximately 4750 ninth grade Algebra 1 students) responses, from 11 high schools and 23 teachers within the district.² The survey, which was developed and validated through previous studies (Anderson, Boaler & Dieckmann, 2018), consisted of 27 questions about mathematics learning with Likert response options. The survey includes (Dweck, 2006) mindset measures (which record between 0.94 to 0.98 in internal reliability and 0.80 for test retest reliability as reported in Dweck, Chiu, & Hong (1995)) along with some focused questions on mathematics teaching and learning. The survey was conducted using the Qualtrics online software and offered in English, Spanish and Chinese. For privacy reasons, the research team was not permitted to directly invite the teachers via email, instead, the district invited all 9th grade Algebra 1 teachers to administer the survey with at least one of their 9th grade Algebra 1 classes.

4.3. Data analysis

The data were analyzed in three distinct phases related to different units of analysis. In phase one district-level survey results were analyzed to look for commonalities and differences across the district. In phase two, qualitative approaches were used to analyze site-level data. Teacher and student interview transcripts were coded in multiple rounds, beginning with open coding to develop bottom-up codes (Emerson, Fretz & Shaw, 2011) and to collaboratively develop a codebook (Saldana, 2015). This codebook was used to code a subset of the interviews, refined through team discussion, and then applied to the rest

² Note: After the initial implementation of this policy, the district began allowing other versions of these pathways with options to “double-up” mathematics courses in 9th or 10th grade (i.e. a 9th grader could enroll concurrently in Algebra 1 and Geometry, or a 10th grader could concurrently enroll in both geometry and Algebra 2 or Geometry and Algebra 2/Pre-Calculus.)

of the interview data. An inter-rater reliability (Campbell, Quincy, Oserman & Pedersen, 2013) of 84% was achieved for the coding of the interviews.

To triangulate emerging themes from the interviews, a similar process was applied to the video data. Content logs of all 16 h of classroom video were developed, which outlined events on each video by time (Derry et al., 2010). Because these logs revealed a wide range of teaching practices, they were then coded by classroom activity to analyze the use of time in each classroom. These classroom activity codes (see Table 3) were developed from a subset of representative content logs, refined through re-watching and discussing videos, and then applied to the rest of the content logs. Coded content logs were then used to calculate the percentage of time each teacher spent on each activity across their two lessons. These percentages were represented visually in the form of a time chart for each teacher, which helped to illuminate differences and similarities in teachers’ use of time.

This phase two analysis of site-level data led to new questions about the curriculum in which teachers and students were engaging. In phase three, three Algebra 1 curriculum units were analyzed—Polynomials, Quadratic Functions, and Quadratic Equations—these units comprise one third of the Algebra 1 curriculum and were the content focus during classroom observations. Each unit is similarly structured with a series of lessons falling in between “Entry”, “Apprentice” and “Expert” level tasks and ending with a summative “Milestone Task” (see Fig. 5). The analysis included each task plus one lesson from each lesson series in the unit. To analyze these tasks, the Nature of Mathematics domain in the Mathematical Mindset Teaching Guide was adapted into a task-based rubric based on two measures—openness of problem(s) and reasoning/multiple perspectives — with indicators for beginning, developing, and expanding ratings (Anderson et al., 2018), see Fig. 6. These attributes of tasks also address the “group worthiness” of the tasks. This rubric was used to analyze a subset of the selected tasks, refined through discussion, and then applied to the remainder of the selected tasks.

Additionally, to connect the curriculum analysis to the observations, all video recordings were coded by unit, lesson series, day/task, and Common Core State Standard, which confirmed that many of the analyzed tasks were observed during implementation. Finally, themes from all three phases of analysis were connected across data sources.

5. Findings

The initial goal of this study was to examine the factors that constrain or support equitable participation in heterogeneous classrooms. Our analysis uncovered several equity focused practices which Louie (2017) deemed as inclusive, as well as several other factors that worked against the principals of Complex Instruction and ultimately, equitable outcomes. The findings of the study come together to tell the story of a district that is experiencing considerable tension. The class-

Table 3
Coding scheme for content logs by activity.

Code	Description
Teacher-led directions (TLD)	Teacher giving directions to the whole class and/or students acting on those directions
Lecture (L)	Teacher lecture, might include Intitate-Resonse-Evaluate (IRE) teacher-student interactions, but no discussion
Whole-class discussion (WCD)	Teachers and students engaged in discussion involving more than one student, which could include turn and talks
Small group (SG)	Students working in small groups with a range of interaction
Independent work (I)	Students working independently, some students may talk, but not collaborating



Fig. 5. Structure of a unit in the district Algebra I curriculum.

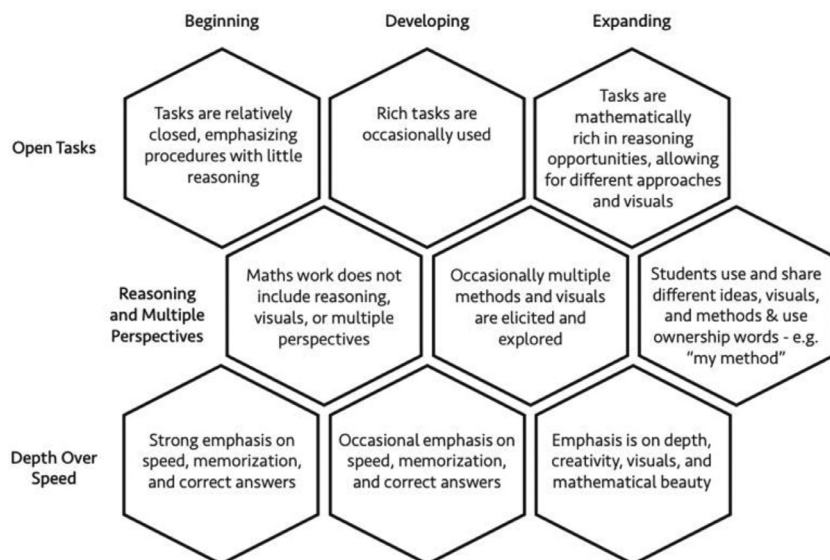


Fig. 6. Mathematical Mindset Teaching Guide (www.youcubed.org/mathematical-mindset-teaching-resources/).

Table 4
Average use of class time.

Category	Average percentage of classtime (%)
Working in small groups	47
Individual work	21
Listening to directions	12
Listening to lecture	12
Whole class discussion	8

room communities showed a strong commitment to groupwork and in interviews students expressed feelings of belonging and a duty to help their peers. However, these strengths were constrained by a procedural, speed-based, and answer-centric approach to mathematics learning driven by the high school mathematics standards. These tensions emerged from the data through five different dimensions that will be examined now.

5.1. Supportive, collaborative classrooms

All teachers in this study had received training through the district in Complex Instruction. Teachers reflected the philosophies of Complex Instruction in their planning and thinking about students as well as the way they organized classrooms. Across the 8 classrooms students spent 47% of class time working in small groups, 21% on individual work, 12% listening to directions, 12% listening to lecture, and 8% in whole class discussions (see Table 4).

Complex Instruction is centered around the idea that no one person in a group will have all the “abilities” needed to complete a task, but that everyone will have some of them, and teachers frequently communicated to students that they were capable of bringing important and meaningful thought to collaborations. Importantly, this sentiment was reflected by all 8 teachers. The following two teacher reflections were typical of the 8 teachers:

“And if we’re doing group work, which we do a lot in here, I expect that they are gonna add a different take to the problem, (...) I expect that they’re gonna add a different perspective than somebody else that might have just been exposed to it.” (Ms. Garcia, Edison HS)

“Everybody has something that they can offer to any type of mathematical work we do together and if willing to put in the work, a tremendous amount of success can happen.” (Mr. Lang, Lewis HS)

Both teachers allude to the differences between students’ prior experiences, but they do not pose these as deficits, instead they describe them as strengths. The students also shared positive ideas about their peers, describing their classrooms as tight knit, non-judgmental and respectful communities:

“I guess we all just kinda respect each other, we don’t assume anything or judge anyone for... I’m not gonna go into race and stuff, I just mean like... We all have like a similar sense of humor, and all that, like... We’re all pretty friendly, no one like starts yelling at the other person, unless it’s like a joke or anything.” (Walker, Turn around, Edison HS)

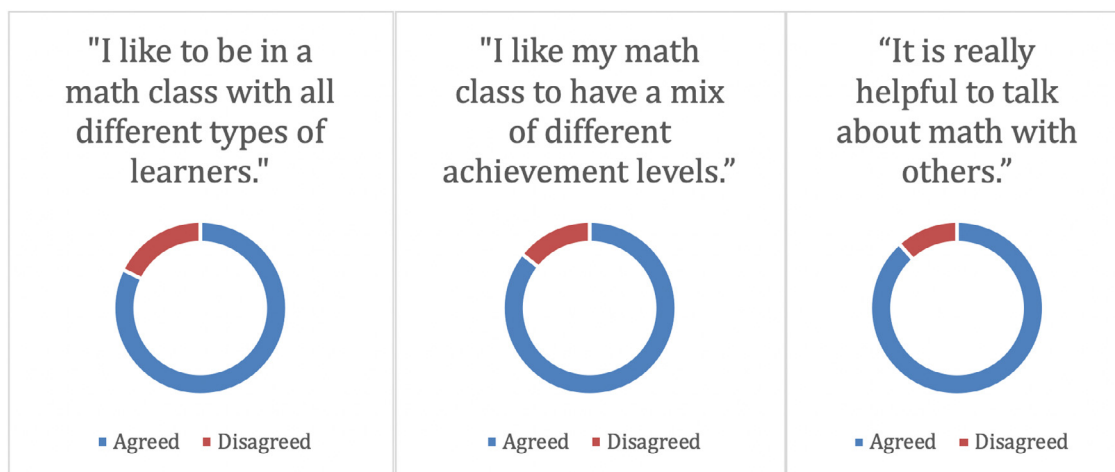


Fig. 7. Results from mindset survey.

"I think everyone's accepting each other. We're like this big community. I feel like I'm accepted there and I feel like I fit in with everyone. They accept who I am and they don't judge me for what I do or how I do it". (Jackie, High achieving, Lewis HS)

District wide, students responded overwhelmingly positively to working with students of different achievement levels in math class. Student surveys ($n = 528$), representing 11 high schools across the district, revealed that 82.4% of students agreed with the statement, "I like to be in a math class with all different types of learners", while 85.8% of students agreed with the statement, "I like my math class to have a mix of different achievement levels". Perhaps most importantly, 88.3% of students agreed with the statement: "It is really helpful to talk about math with others" (Fig. 7). Each of these results show that students value and enjoyed working together with peers in heterogeneous classrooms.

Another key norm of Complex Instruction is that everyone has the right to ask for help and the duty to give help when asked. This norm was consistently reflected in the ways that students described their experience working in groups. The practice of supporting peers in small group work came up in all but one of the twenty-two student interviews. In fact, students expressed that they felt duty to help their peers simply when they appeared to be struggling. For example, Teresa and Mario (both high achieving from Lewis HS) shared:

Teresa: Now, I see that a lot. If they didn't ask for help but they see them struggling, they're gonna be asked, "Do you need help?"

Interviewer: Even if they don't ask?

Teresa: Yeah, even if they don't ask. They could like, they would just see like, 'she's still on like question one. I should help her,' and they would help.

Mario: Cause sometimes the person's too shy to ask for help, or like maybe they think they are not good enough to ask for help.

Interviewer: Does Mr. Lang ask you to do that?

Mario: No.

Interviewer: Why do you do that?

Mario: Well 'cause... [I] do it because I wanna help people. I don't want to just be selfish and look at them struggle while they work. I actually want to help them and that's why I do it.

Teresa: I agree with that. 'Cause I used to be one of those people who would struggle a lot back then and I didn't like how people wouldn't help me at all. So I was like, 'Oh, since now that I actually under-

stand, I should help them so they wouldn't feel like I did.' (Teresa & Mario, high achieving, Lewis HS)

Another student, at Edison HS explained:

"If I just see them... I feel like if they gave up, I would definitely help them. But if they told me, I would of course help them. The way I see it, someone needs help, is like if they're just tapping their pencil or just re-writing the same thing over and over, I would ask, "Hey, do you need help? Are you stuck on something?" (Kaitlyn, uncategorized, Edison HS)

The students' commitments to help others were admirable, and reflected the equitable focus provided by the teachers, but other sources of data revealed that the strong community norms were weakened by other dimensions of the classroom approaches.

5.2. Classroom structures working against equity

Complex Instruction recommends that students stick together as they work through tasks with the norm that no one is finished until everyone is finished – a principle that is based upon the use of broad, multi-dimensional tasks that students can encounter in many different ways. At Gateside, this norm was enforced through the use of 'checkpoints' – a particular place in the mathematics problems when students were meant to call the teacher over to check on their work. If any group member had not completed all of the work leading up to the checkpoint or not reached the same answers, the group would not be approved to move on until the group had helped all students reach the same place. In several of the classrooms, successful checkpoints resulted in the achievement of a stamp, and in some cases, students were expected to earn a certain number of stamps in order to receive classwork credit for the day. Stamping is not a recommendation in Complex Instruction (Cohen & Lotan, 2014, p. 66) and seems to have developed as a practice in only some CI classrooms.

Walker from Edison HS explained this dynamic and the ways his group would work with students who did not understand:

"She told us from the beginning of the year, "If one person isn't done..." then she won't stamp all of you. So, if you have a group of three everyone needs to be done with the same question. Group of four, same situation. So if someone gets stuck then we go... We normally we just send, not send... We just have one person explain 'cause having three people explain at once is kind of hard to keep up with." (Walker, Turn around, Edison HS)

Walker emphasizes the importance of all group members being finished at the same time in order to receive stamps. Some of the students

reported that this practice made them feel pressured – as Ashley, a lower achieving student described:

“Well, for us, we do stamps, and the whole group has to be in the same question to get a stamp, and if some people just do their work and start rushing the person that’s on the first question and they’re almost done, and they’ll be like, ‘Oh hurry up. We need a stamp, we need a stamp,’ and we have to hurry up for *them* to get a stamp.” (Ashley, Low achieving, Edison HS)

CI recommends that students work at the same pace with the assumption that the work they are doing is multi-dimensional. At Gateside the majority of the work was procedural, and when this was combined with a practice of stamping, it resulted in several negative outcomes. In these ways the classroom practices diverted from the recommendations of CI, a finding that emerged as critical in the teaching and learning that ensued.

The need to work at the same pace and to receive stamps contributed to a high proportion of students in Gateside district who believed that successful mathematics work meant working at speed. In surveys 80% of students agreed with the statement: “People who really understand math will get an answer quickly.” The idea that mathematics success comes from fast working is a damaging and limiting belief that often harms students’ mathematical progress and causes students to de-identify with the subject (see also Boaler, 2019; Boaler et al., 2019)

The need to work at the same speed seemed to cause tensions, not only for the students who believed they were holding up the progress of their group, but for the students who wanted to move on. Theo, one of the high achieving students, explained that he sometimes felt “hopeless”:

“It’s just you help out your classmates. It’s like if they don’t get it, you can help them understand the subject. And if you don’t get it, you can ask the teacher. So it’s like that’s kind of how I help my classmates. But sometimes it just feels like the person won’t understand the subject or is not interested in understanding the subject, and that really makes you feel hopeless, especially when that method of assessing people is kind of formatted in a way in which you cannot really have any control over someone else’s work.” (Theo, High achieving, Edison HS)

The tensions were not only experienced by high achieving students, the students who needed more time also explained that the process of needing to be finished together made them feel uncomfortable:

“I don’t really like asking them questions ’cause they’re like, “You still don’t know this yet?” Stuff like that.” (Jose, Low achieving, Lewis HS)

“Well, I personally prefer working by myself ’cause it feels weird when... I normally get stuck a lot in math, so I’m just like... I feel like when I’m alone, I can ask a teacher, I don’t have to ask other teammates. And that it’s easier too for me to ask a teacher.” (Silvia, Turn around, Lewis HS).

The pressure students felt to work at the pace of the group may have contributed to the numbers of students believing that speed was important and the 53% of students who agreed in the student survey that “It is important not to make mistakes in math.”

5.3. Narrow mathematics

In previous studies of students working in heterogeneous classrooms, employing Complex Instruction, equitable outcomes were reached through the multi-dimensional mathematics that students experienced. As discussed previously, the success of a Complex Instruction approach rests upon three central principles, one of which is the presence of multi-dimensional and group-worthy content. If content is narrow it undermines the idea that all students have different strengths and

Table 5
District curriculum analysis results.

Category	Criterion	Percent (%)
<i>Beginning</i>	Tasks are relatively closed, emphasizes procedures with little or no reasoning, do not include visuals or multiple perspectives	55
<i>Developing</i>	Tasks emphasize procedures with reasoning, multiple methods and visuals are sometimes elicited and explored	45
<i>Expanding</i>	Tasks are mathematically rich in reasoning opportunities, allow for multiple approaches and visuals, students use and share their own methods	0

severely weakens the purpose and need for collaborative work. If content is multi-dimensional - emphasizing student thinking and reasoning, through multiple representations of mathematical ideas - then different students can bring contrasting strengths to the work and all group members are needed to complete the task. In this way, all students are positioned as resources for one another in the learning process, rather than students being positioned as those who give help and those who receive it. In mathematics, multi-dimensional work usually means open problems that ask students to communicate, reason, visualize, draw, and build— instead of narrow questions emphasizing one method with one answer.

In Gateside district we expected to see students engaged in reasoning about mathematical ideas but instead found that students were mainly focused upon reproducing methods with one answer. In student surveys 94% of students agreed that “Mathematics involves mostly facts and procedures that have to be learned.” In interviews students revealed that their worthy motivation to help others was focused upon guiding their peers to “the answer”:

“We help them, or we ask, “Are you stuck?” And if they say, “Yes,” Then we guide them to the **answer**.” (Robert, High achieving, Edison HS)

“We usually help them, like we’ll be like, “Oh, so here’s the **answer**,” and then sometimes I’ll just be like, “You know, let me show you,” and then I just do it for him.” (Karen, uncategorized, Edison HS)

Classroom observations and student perceptions (from surveys and interviews) both suggested that the majority of classroom time was spent working on narrow mathematics. This led to an expansion of our research focus beyond classroom interactions to the curriculum used in the different schools – which was provided by the district.

A combination of 20 lessons and tasks from 3 curriculum units were rated by 2 team members giving the results in Table 5. This shows that 55% of tasks were rated in the beginning category, and 45% as developing - with no tasks rated as expanding. Fig. 8 gives examples of algebraic tasks rated as beginning, developing and expanding:

In addition, the vast majority of the curriculum that fell under the “developing” category were those tasks created or chosen by the teachers, whereas the majority of textbook tasks fell under the “Beginning” category.

The content of the lessons studied was algebra – in particular functions and expressions. These areas of mathematics could be described in content standards in multi-dimensional and connected ways. Instead they are presented in the Common Core standards as individual and segmented areas of knowledge (see Table 5). For example, under the topic of “Algebra” and subtopic “Seeing Structure in Expressions”, Standard A.SSE.3 emphasizes students working with equivalent forms of expressions and naming properties of expressions. This subtopic could highlight multiple forms of algebraic expressions, including visual patterns, models and drawings. Instead the wording of the standard emphasizes

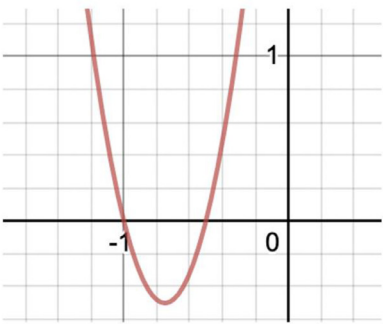
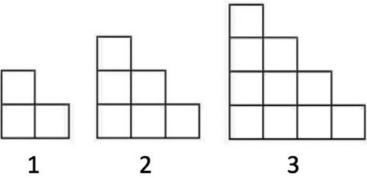
Beginning	Developing	Expanding
<p>Solve each quadratic equation:</p> <ol style="list-style-type: none"> $(2x+1)(3x+4)=0$ $(2x-7)(x+2)=0$ $(5x-9)(4x-6)=0$ $(4t+3)(2t-5)=0$ 	<p>What are the zeros? Prove it algebraically.</p> $y = (2x+1)(4x+4)$ 	<p>How do you see the pattern growing?</p>  <p>(leading to a discussion of different, yet, equivalent expressions)</p>

Fig. 8. Example tasks by rating.

the expression itself, communicating a focus on procedures and notation rather than multiple representations. Similarly, Standard A.APR.1 communicates a focus on the algebraic manipulation of polynomials but does not express conceptual understanding or reasoning nor connection to the purpose or use of polynomials, or connections to other areas.

Standards F.IF.7 and F.BF.3 together fall under the CCSS topic of functions. The concept of ‘Functions’ has the potential to work as a big, connected idea. The series of standards could work together to build out a deep and multidimensional approach to learning and understanding the behavior and use of functions. Instead the standards focus on a one-dimensional approach to graphing functions by hand and naming key features. The standard describes comparing values in F.IF.7 and naming characteristics of graphs based on algebraic manipulations of the functions in F.BF.3. None of these descriptions require mathematical thinking and reasoning beyond memorizing a procedure.

Mathematical practice standards, that encourage reasoning, collaborating and active engagement, are given alongside the content standards in the Common Core. The inclusion of mathematical practices in state standards was considered a major step forward when the standards were released (Bennett & Ruchti, 2014; Johns, 2016), unfortunately, the content standards at the high school level are so numerous and discrete, many teachers find implementation of the practices too challenging.

Our study of the high school algebra standards that are used by textbook writers, districts, and schools, to guide instruction, found that the mathematics was conceived and communicated as a narrow and disconnected subject. The impact of this narrow conception of mathematics was seen in classrooms as students were invited to engage in closed questions, with few opportunities to engage in reasoning, connection making, or deep understanding. These observations were made in a district that has committed to a more engaged and equitable teaching of mathematics. The narrow mathematics content emerged as an important factor working against equitable practices and engaged students, leading instead to narrow conceptions of student ability.

5.4. Binary perceptions of math ability

While the teachers and students both reflected equity minded sentiments about learning together in an inclusive environment, there remained a hierarchy of the abilities that were valued in practice. In surveys about a third of students held fixed ideas about math ability with 34% agreeing with the statement: “There are limits to how much people can improve their basic math ability” and 36% agreeing that “People can learn more math, but they cannot really change their basic intelligence.”

The interviews overwhelmingly confirmed these ideas about mathematics ability, which seemed to derive from the students’ ideas that mathematics learning is about speed and correct answers.

For example, Albert, from Edison HS saw himself as being “better” at math than his peers because he was “quicker:”

“I would say I’m better at math than they are, because I usually understand the equation quicker than them. ‘Cause when somebody explains it to me and shows me how to do it, I usually remember how they did it. So then, I understand it faster, and then I help out my classmates if they didn’t get it, didn’t catch how to do it. So that’s why I think I’m maybe higher at liking math than they are, or achieving, getting better grades easier...” (Albert, High achieving, Edison HS)

The focus on speed and answers led to frustrations from the students who were not as fast as other students, and led to feelings of inadequacy.

Lauren: I get frustrated, I just go like, “Why can’t I do this?” And I’m like, “I have to ask people I don’t like know.”

Interviewer: Would you tell me more?

Lauren: So I get infuriated ‘cause I feel like the smart people in the class are like, “You’re so stupid, you can’t do this,” and I’m like (thinking to herself), “Why don’t you get it?” (Lauren, Low achieving, Edison HS)

Overall, the students described a general divide between the students in their classrooms who were quick and slow, and who found mathematics “easy” or needed help. Mario, a high achieving student at Lewis HS, describes this perceived divide between students in terms of mathematics ability, also revealing the unfortunate perceptions of mathematics exclusivity and “genius” that have been shown to lead to inequitable participation and the exclusion of women and students of color (Leslie, et al., 2015; Chestnut et al., 2018).

In the beginning of the school year, I thought I was gonna be the smart one in the class. It turns out there’s this another student, that’s a girl, and she’s studying math by herself, and she is like exceeding my expectations of what a math genius is, you know. So, I think there are different levels of math knowledge. Because some people need help, and some people don’t. Some people find it easy, and some people get well what math is going to and... well, you know, like that. (Mario, High achieving, Lewis HS)

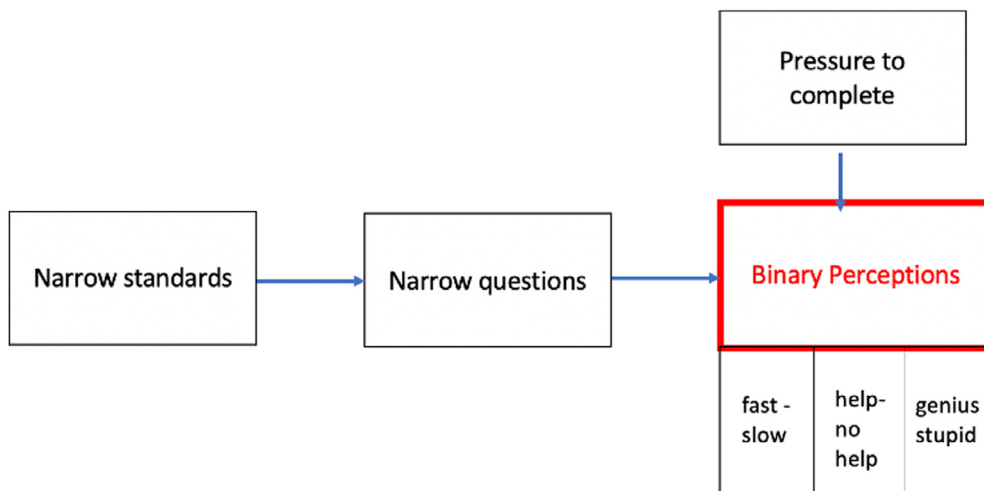


Fig. 9. The Development of Binary Perceptions of Students.

Teresa concurs with the binary perceptions of student ability, explaining that students in the class fall into one of two categories. Similar to other students, Teresa's categorization of people draws from the speed at which they work:

There's a range but there's like a few people out of the range, yeah... I guess two different groups you could be in. I separated into a category of groups; there's this one group and then there's this other group. There's a group where you could be with the people out of the range and sometimes they would go ahead of you. And then yeah, I would be too shy to ask them cause, like, they're already on question like five and I'm on question two or something. And then there's could be like a group where we all stick together. But even while with the group that leaves you behind, I feel like I just needed to ask them and they would, if I did ask. (Teresa, High achieving, Lewis HS)

In interviews with 41 students the majority of them shared beliefs that their peers are divided into distinct groups. Students describe these groups using various binaries such as those who get it and those who do not, those who are fast and those who are slow, those who need help and those who do not, the smart ones and the stupid ones. This study has found that the binary perceptions were initiated by narrow conceptions of mathematics which worked against equitable outcomes (see also Fig. 9).

6. Discussion and conclusion

Gateside District is an unusual school district in many ways. It is an urban and diverse district and one of the largest in California. More notably, it is a district that is committed to equitable outcomes (Ellis & Berry, 2005; Gutiérrez, 2017; Hand, 2010; Joseph et al., 2017) and one that has been prepared to study research, act on the findings, and make hard decisions even in the face of public opposition from groups of parents. These hard decisions have included district wide de-tracking (Oakes, 1986; Boaler, 2017, 2013, 2008; Boaler & Staples, 2008) and the teaching of algebra in ninth rather than the more currently popular eighth grade (Daro & Asturias, 2019). The commitment of the district to high and equitable mathematics outcomes has also included wide-scale professional development for teachers and dedication to the principles of Complex Instruction (Cabana et al., 2014; Cohen & Lotan, 1997). These changes have brought about impressive outcomes, with algebra failure rates declining dramatically and large increases in students taking advanced mathematics courses.

This study set out to investigate the factors working to support or constrain equitable outcomes. Observers walking into the algebra classrooms of Gateside district would probably notice hard working teachers

and students, but a deeper study and analysis, drawing from multiple forms of data including classroom observations, interviews with teachers and students and survey results, highlights important factors that work against equitable outcomes and seem important to understand, for these and other teachers working towards equity (Cabana et al., 2014). Our analysis centralizes student voice, as the students themselves give the greatest insights into their own learning and ways of approaching mathematics and collaboration, that were supported by other data. Their words helped unlock important new knowledge, giving insights into the ways equitable progress can be diminished and the factors at work in this process.

Importantly, the algebra classrooms of Gateside district are made up of supportive communities of learners who care about each other and respect each other's thinking (Boaler & Staples, 2014). Students regard helping other students as part of their classroom mission. This came from the work of teachers spreading the messages of Complex Instruction (Cabana et al., 2014; Cohen & Lotan, 1997) – communicating that all students are valued, that students have a duty to help each other, and that there are many ways to be successful. Unfortunately, the achievements of this work – which were extremely important – were held back by other factors, that ultimately caused student stress and the development of negative ideas about other students and about mathematics learning.

One of the factors came from a particular instructional strategy that many of these teachers used to implement Complex Instruction. A central element of Complex Instruction pedagogy is the recommendation that students “stick together” – with an assumption they will do this through reasoning, visualizing and thinking together (Cabana et al., 2014). In the Gateside classrooms, students were expected to work on closed questions together, finish together, pass a check-in and receive a stamp. While the norm “no one is done until everyone is done” might make sense when students are working together on multi-dimensional tasks – sharing reasoning and making connections, when the work is instead focused on finding right answers, being “done” essentially means having the same answer on everyone's paper. This process caused considerable tensions for the students and served to communicate that speed is an indicator of mathematics success, an unfortunate message that the teachers did not, themselves, believe. Classrooms will always include students who work at different rates, and the requirement to finish work together will always prioritize a “group rate”, leaving slower students, often those who think more deeply or creatively, to feel that they are holding back their group. The variation in student speed is mitigated when tasks value different types of thinking and teachers emphasize that mathematics is about depth and connections rather than fast finishing (Boaler et al., 2019). In the absence of multi-dimensional mathematics, the norm of finishing together was problematic.

The requirement to work together and receive a stamp is a teacher directed practice that could also convey a lack of authority and responsibility for the students themselves. Cohen and Lotan (2014) specifically warn against extrinsic rewards for compliant behaviors as this works against the principals of CI. However, it seems some mathematics teachers have adopted this practice, to help manage groupwork. When high school students feel that they cannot move on until their teacher has “checked them off” it reduces their own development of autonomy and self-responsibility, both of which are important for adolescents and have been associated with high and more equitable achievement (Black & William, 1998, 2005; White & Frederiksen, 1998). We recognize that the CI recommendation for group finish and teacher check was driven by important principles of equity, including student accountability and equal participation. But this study revealed that this practice, paired with the absence of multidimensional and groupworthy content, (Cabana et al., 2014) worked in counter-productive ways and that different requirements and messages could be more helpful for the work of teachers and students.

High school (and younger) students are extremely capable of determining when work is valuable for their understanding and when it makes sense to move on (White & Frederiksen, 1998), and it is entirely possible to communicate to students that they help each other when needed, without the requirement that they all finish together, or that teachers validate their progress. A practice that could replace the group pace and teacher check in, could be students being given a rubric communicating the goals of learning, and time for self-reflection, to decide which learning goals have been achieved (Boaler, Dance, & Woodbury, 2018; Brookhart et al., 2016). A rubric for the students could include the goal of working productively with others. This would remove the pressure to finish at a certain speed and replace the practice of teachers assuming responsibility for “checking” on learning with students developing responsibility for their own learning, and the important learner autonomy that comes with such responsibility (Boaler, Dance, & Woodbury, 2018; White & Frederiksen, 1998).

Group work is extremely important to mathematics learning, as students connect ideas they increase their understanding and engagement (Boaler, 2019a; Cabana et al., 2014). Complex instruction has been developed to help groupwork be more equitable, and many features of the approach were highly successful in the classrooms observed – features such as recognizing the strengths of all students, “assigning competence” and group roles (Velazquez & Louie, 2014). The additional feature of students staying together and checking with teachers to move on seemed problematic and not a necessary part of equitable groupwork. In other examples of equitable groupwork approaches, students are given more open tasks and encouraged to take them in directions that interest them. All students work together and help each other and when it is time to report out, they are interested in the work of other students that may be different to their own (see also Boaler, 2019a, b). We propose that an update to Complex Instruction might be one that encourages student responsibility and assessment through learning rubrics (Black & William, 1998, 2005; Boaler, Dance, & Woodbury, 2018) as well as focused communication to CI schools regarding the central role of multi-dimensional mathematics.

A second, related factor that was highlighted in this study was the development of binary perceptions of people, who were believed by other students to be fast or slow, needing help or not, and even gifted or stupid. These negative binary perceptions were encouraged by the practice of group finish, but they also derived centrally from the mathematics students were learning. When mathematics is narrow and questions value the completion of procedures at speed, students develop narrow perceptions of what it means to be a valuable mathematics learner. The connection between the nature of student learning and the nature of student perceptions emerged clearly from the data. In this study the students’ conflicting ideas about the value of working with other learners, at the same time as thinking in binary ways about each other, caused them stress and frustration, that they clearly communicated to interviewers.

Two of the binaries that students developed caused them to believe that working slowly, and needing help were both negative attributes of mathematics learners. Yet the converse of these ideas – that working slowly and deeply is a critical part of mathematics, and struggle is an important time of learning, that develops brain connections and growth – are important ideas for learners that have been shown to boost learning and achievement (Anderson, Boaler, & Dieckmann, 2018; Boaler, Dieckmann, Pérez-Núñez, Sun, & Williams, 2018). Struggle should be celebrated – everyone struggles, and it is unhelpful for students to divide each other into those who do and don’t struggle and need help. This binary perception causes students to fear times of struggle and of making mistakes, when both are critical to effective learning. Similarly, slow learners are some of the deepest and most important mathematical thinkers (Boaler, 2019). The need to finish narrow questions at a certain speed and receive a stamp works against the important message of working slowly and deeply. Many students communicated that their inability to keep up with their group led to feelings of inadequacy as mathematics learners. The practice of group finish and the nature of the mathematics in which students were engaging both worked against the communication of messages that could stay with students for many years.

In addition to the problems raised by group finish and teacher validation, the equity goals of the district, supported by a range of good practices, and by well trained and equitable teachers, were being undermined by the mathematics on offer to students. There was a direct relationship between the narrowness of the mathematics and the development of binary perceptions of mathematics achievement, based on fixed ideas, speed and lack of struggle. All three of these ideas – of fixed brains, avoidance of struggle and the need to work quickly - are harmful to learners, whether they are students categorized by others as successful or unsuccessful (Anderson et al., 2018).

Narrow mathematics has prevailed for centuries in the US (Boaler, 2015), this study gives some insight into the sources. Past recommendations for change have focused upon working with teachers, improving tasks, and eliminating tracking. Gateside district, admirably, had enacted all of these practices, but the narrowness of algebra standards had led teachers to view mathematics as a set of isolated methods, which undermined their important work. As the nation moves to a greater awareness of the need for more equitable structures, we encourage the gaze in future work and in policy to fall upon the narrow mathematics standards that are so important to every student’s future. Districts such as Gateside have made tremendous progress but their important equity focused work is restricted by the narrowness of the mathematics communicated to leaders and schools. Mathematics is a rich, varied and multi-dimensional subject. Algebra students can engage in important work, reasoning, connecting, developing their ideas. It should not be left to our nation’s most inspirational teachers to take the narrow conception of mathematics set out in the standards and translate it into multi-dimensional and engaging content – and assessment (Black & William, 1998, 2005; Boaler, Dance, & Woodbury, 2018). There is evidence that teachers and districts can do this work (Anderson et al., 2018), but such examples are rare, and all teachers should be encouraged to engage students in important mathematical practices.

It is typical in research to locate the source of narrow content with classroom activities, with calls for more engaging tasks. But the endurance of narrow algebraic experiences (Demosthenous & Stylianides, 2014), and the preponderance of narrow questions in textbooks (Park, 2011) even after the implementation of the Common Core standards, led us to consider the mathematics content standards that are the foundation for most district curriculum decisions. The narrowness of the high school mathematics standards (as shown in Table 6) leads many districts and publishers across the United States to present mathematics to students as a set of narrow questions that emphasize one method and one answer. This is despite the calls of mathematicians to share mathematics as a vibrant and connected subject (Strogatz, 2012, 2019; Wolfram, 2020). This study has shown that when mathematics

Table 6
Selection of Common Core State Standards covered by district curriculum.

Topic	Subtopic	Example Standard
Algebra	“Seeing Structure in Expressions”	A.SSE.3 Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.
	“Arithmetic with Polynomial and Rational Expressions”	A.APR.1 Understand that polynomials form a system analogous to the integers, namely, they are closed under the operations of addition, subtraction, and multiplication; add, subtract, and multiply polynomials
Functions	“Interpreting Functions”	F.IF.7 Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.
	“Building Functions”	F.BF.3 Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.

is taught as a narrow set of methods, this works against equitable outcomes that teachers and districts value highly and work hard to achieve. The narrowness of the mathematics being taught leads to a binary perception of student ability and a focus on answer finding. When there is one right approach, one right answer, and classroom practices encouraging completion at a set time, students become sorted into binaries of those who can and those who cannot.

We end this paper with a call for a revision of the Common Core Mathematics Standards. When the Common Core was introduced there was widespread recognition that the K-8 standards had improved mathematics but the high school section was largely unchanged from previous narrow standards adopted in various states. The addition of the mathematical practices has not proved to be impactful, probably because they are seen as separate and incompatible with the narrow content standards. A revision of the Common Core mathematics standards could include integration of content and practices in new descriptions of mathematics that emphasize its connected and coherent nature. Revisions should also include consideration of the most important mathematics content that students should learn, as the content of high school mathematics classrooms has not substantively changed since the 1800’s, despite the changing nature of the mathematical needs in the world (see also Boaler & Levitt, 2019; Strogatz, 2019; Wolfram, 2020; Rockmore, 2020). These changes would directly support teachers and administrators in their pursuit of more equitable teaching and learning. The results of this study should inform mathematics revisions as it has shown that the narrowness of the high school mathematics standards leads directly to binary perceptions of achievement. Multi-dimensional content, by contrast, may be key to the equitable outcomes that districts, administrators, and teachers work hard to pursue and are foundational to the goals of the United States.

Declaration of Competing Interest

The authors declare that they have no known competing financial interests or personal relationships that could have appeared to influence the work reported in this paper.

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