Dynamic assessment and teachers' knowledge of children's mathematical thinking: a case study in children's mathematics

LIO MOSCARDINI and CHIARA MOSCARDINI

This paper considers the kind of pedagogical knowledge and principles involved in the operationalisation of knowledge of children's mathematical thinking as a process of dynamic assessment. Using a case study of a particular child, this paper explores planning and instruction for a child determined by a detailed and informed interpretation of the child's conceptual understanding through a dynamic process. It presents as a case study the observations of a teacher who had undertaken professional development in children's mathematical thinking, theoretically informed by Cognitively Guided Instruction and Maths Recovery. The observations revealed the child's mathematical understanding and how the teacher used this knowledge dynamically to inform teaching. The paper outlines the kind of knowledge required of teachers to enact this dynamic process in mathematics teaching and argues for the centrality of this to the development of inclusive practice.

***Key words*: dynamic assessment, Cognitively Guided Instruction, children's mathematical thinking, inclusive practice.**

DOI: 10.1111/1467-9604.12331

Introduction

Understanding the details of children's mathematical thinking is central to supporting them in their learning (Wright *et al*., 2000; Empson and Jacobs, 2008; Walshaw, 2012). In mathematics education, the relationship between teachers' knowledge of children's mathematical thinking and how this knowledge comes to be applied in relation to particular children is a dynamic process of assessment and instruction which potentially supports the inclusion of all learners through recognising and responding to particular learners (Moscardini, 2010; 2014; Hunt and Empson, 2014). Teaching mathematics requires specialised knowledge (Hill *et al*., 2008; Thames and Ball, 2010); supporting children's learning in mathematics through their own sense-making requires the operationalisation of that specialised knowledge and teachers' engagement with children's mathematical thinking through a dynamic process (Jacobs *et al*., 2010). While dynamic assessment is usually seen as the province of educational psychologists, some educational psychologists, particularly those who seek to promote more inclusive forms of practice, have called for a greater consideration of the child within the learning context of the classroom (Farrell and Venables, 2009). This paper considers the role of the teacher in this process.

Models of support in inclusive practice require moving on from the idea of identifying a child as failing and then seeking some kind of remediating response towards a more preventative position where a teacher can recognise and act upon a child's understanding dynamically (Franke and Kazemi, 2001; Elliott and Resing, 2015). Common and arguably unhelpful approaches may involve an intuitive response (Moscardini, 2015) or focus on curriculum delivery and the achievement of narrowly set targets within a lock-step process (Goepel, 2009; Sanches-Ferreira *et al*., 2013). Recognising and responding effectively to a child who may need support in a way that is non-stigmatising, inclusive and respectful of the child require knowledge of the child's mathematical thinking along with an understanding of the funds of knowledge which the child brings to the learning experience (Turner and Drake, 2016). This is not a simple matter. It requires the enactment of pedagogical practices supported by a teacher's domain specific knowledge and pedagogical knowledge for structuring and organising teaching (Shulman, 1986) along with the mobilisation of a deep knowledge of children's development and thinking in specific domains (Ball *et al*., 2008). It involves supporting the development of all children while at the same time responding at the level of individual children (Lewis and Norwich, 2005). Crucially, it requires an understanding of how individual children are making sense of particular concepts (Jacobs *et al*., 2010) and understanding not simply 'whether' a child has learned a particular aspect of what is being taught but gaining an insight into 'what' conceptualisations and misconceptualisations (or perhaps preferably, the partial understandings) that a child has made (Torrance and Pryor, 1998, p. 153). In relation to assessment practice, this proposition places teachers at the heart of the process.

Dynamic assessment

Dynamic assessment covers a range of approaches and has been described as an 'umbrella term' (Elliot, 2003 p. 16) to outline an approach to assessment that connects the assessment to learning and teaching processes (Lidz and Elliot, 2000; Elliott, 2003; Grigorenko, 2009; Lidz, 2014). Historically, dynamic assessment has been considered from the perspective of the educational psychologist (Elliot *et al*., 1996; Stringer *et al*., 1997). However, increasingly it is being seen in relation to the classroom with recognition of the role of the teacher in this process (Farrell and Venables, 2009; Stringer, 2009).

Munn (2008) described dynamic assessment as having one key function: 'to ground the teacher in the learner's understanding and to direct subsequent teaching' (Munn, 2008, p. 5). A fundamental feature of dynamic assessment is the interactional basis in which the assessor actively explores the learner's thinking processes. Its aim is to generate a response to support the growth and development of the learner and is informed by what can be learned rather than what has been learned (Lidz, 1995; Elliot, 2000; Munn, 2008; Grigorenko, 2009). Munn (2008) described dynamic assessment as a conversation to examine knowledge as process rather than the product of learning and as being more closely and dynamically linked with teaching than with other forms of assessment. However, consideration needs to be given to the nature and quality of knowledge required to facilitate this process.

Calls for a closer alignment of dynamic assessment with the practice of teaching have been made because of the potential usefulness to teachers and psychologists in developing classroom-based educational interventions (Elliot, 2003; Stringer, 2009). This dynamic and potentially transformative activity acknowledges the assessment process as embedded in teaching acts. Drawing from a rich case study which provides a contextual basis, this paper focusses specifically on the role of the teacher and considers the relationship between a teacher's knowledge of a child's mathematical thinking and a process of assessment and instruction and how these might be aligned to dynamic assessment.

Aims

The aim of the study was to focus on how a teacher used her knowledge of children's mathematical thinking to dynamically assess and instruct a child. Using a case study of a particular child, the paper explores planning and teaching determined by a detailed and informed interpretation of the child's conceptual understanding through a dynamic process and considers the relationship between a teacher's knowledge of a child's mathematical thinking and engaging in a process of assessment and instruction that can be aligned to dynamic assessment.

Method

By focussing on the mathematical engagement of a child within the real-life context of the classroom, the study demonstrated the three key attributes for a case study outlined by Merriam (1998) as particularistic, descriptive and heuristic. It was particularistic in its focus on a particular child specifically in the area of her mathematical understanding; descriptive in providing a clear and rich account of her mathematical sense-making and heuristic in giving an insight into this process (Merriam, 1998, p. 16).

Analysis followed a process of direct interpretation (Stake, 1995) informed by the theoretical frameworks of children's mathematical development provided by Cognitively Guided Instruction (CGI) and Maths Recovery (Wright *et al*., 2000). Although case studies are not readily generalisable to wider populations, the value in a case being driven by a theoretical proposition is that it allows the case to be generalised to this proposition (Yin, 2003).

Ethics

The study fulfilled the requirements of the University of Strathclyde School of Education Ethics Committee. This was the institution with which the lead author was affiliated at the time of the study. The study was carried out in the context of regular and scheduled school-based collaborative practice and conformed to the school and Local Authority policy and guidelines. Participants gave informed consent, they have been anonymised to ensure that they, the school or Local Authority cannot be identified.

The case study

Background

The case study is set in the context of the work of a support teacher who is an experienced CGI practitioner and who had also undertaken professional development in Maths Recovery. The support teacher's role, as determined by the Local Authority remit, was to support the class teacher in developing numeracy within the classroom. Through observations and discussions with the class teacher, it became apparent that there was a significant gap in the mathematical understanding of a number of children in the classroom in relation to the mathematics tasks set. The support teacher worked in collaboration with the class teacher with all of the children, however, one child, Lara (pseudonym), is presented as the central focus of this particular case study. The work with Lara provided a rich body of data which was used iteratively with the class teacher to highlight and respond to her conceptual understanding.

Profile of the child

Lara is an eight-year-old in primary 3 in a school in an area of high socio-economic deprivation. A standardised mathematical assessment conducted across the local authority identified 11 out of 17 pupils in the class to be in the bottom 5% of all learners, with a further six pupils in the bottom 5%–15% of learners. In these assessments, Lara scored the second lowest in the class. Lara's mother had previously expressed concerns to the school about Lara's understanding of numeracy. Lara was a quiet and undemanding child, however, as there were a number of children in the class who presented difficult and disruptive behaviours the teacher's attention was frequently focussed on these children. A preliminary classroom observation by the support teacher found that Lara copied her answers from others without attempting problems herself.

Current planning and classroom experience for Lara

Lara followed the general curricular planning for mathematics in the class. She received no additional support and there was no individualised planning provision in place for her. There was no suggestion of further support being provided. The mathematics planning for all children in Lara's class, including Lara, was to work on vertical addition of double digits with exchange. The school followed a commercially produced mathematics scheme which placed a strong focus on whole class 'mental maths' with children expected to solve mathematical calculations without materials and focussing on the direct recall of number facts. Teaching was usually done as a whole class with the teacher demonstrating the process on the Smartboard and pupils writing answers on whiteboards.

Observations and data collection

The collaborative sessions were part of an educational support initiative implemented by the local education authority. These sessions involved a support teacher working collaboratively along with the class teacher to provide classroom-based support. The sessions were one-hour sessions, twice weekly over a three-month period. They took place in the regular classroom with the support teacher working alongside the class teacher. The sessions were usually whole class or groups and frequently led by the support teacher although this was decided collaboratively. It was within this context that the support teacher carried out detailed observations of Lara attending to a series of problems. The findings presented are drawn from three successive 30-minute sessions. During these sessions, Lara was free to solve problems in any way she chose and a range of materials were at her disposal including whiteboards, pens, paper and counting materials such as cubes. Data gathered were the support teacher's field notes and records of Lara's solution strategies.

Observations

Observations of Lara's strategies relating to counting, addition and subtraction problems and algebraic reasoning are presented. Some tasks provided similar information regarding Lara's understanding and so her solution strategies to only some of these activities are illustrated below. The observations are taken directly from the support teacher's records and are a descriptive account of interactions. The observations are not all sequential, they are extracts from three separate sessions. Some of these observations are taken directly from the field notes and are in the first person. These observations are followed by a theoretical analysis.

Counting – number word sequence

Lara could recount a number word sequence from 1 to 50 confidently. She was able to go back from 10 to 1, although with some pausing in between numbers. Counting back from 20 Lara counted '20, 90, 80, 70, 60' then stopped and said, 'that's all I can count to.' Lara was able to recount a Number Word Sequence from 7, stopping at 50 when asked. When asked for the number before 15, she replied, '51'. When asked for the number before 19 she said 20. When asked for the number before 12 she started counting from 1 to 12, then stopped and said, 13. When asked for the number before 13 she said she did not know.

Additive tasks

Lara was presented with an unscreened addition task: a set of three cubes and a set of two cubes on the table and asked how many cubes were there altogether. Lara looked at them and said, ‘five’. When asked how she did this she said, ‘I started at one and went all the way up to five.’ When presented with different sets of cubes (5 + 4; 8 + 15) Lara started at one each time and counted all the cubes together, nodding as she counted.

Lara was presented with the word problem:

> Andrew is playing football. He scores 2 goals in the first half and 9 goals in the second. How many goals did Andrew score altogether?

Lara wrote the numerals 2 and 9 on her whiteboard. She then wrote 2, 3, 4, 5, 6, 7, 8 and 9 above it. Lara then counted from 2 to 9, starting with the number word, 2. She then wrote 29 and said that he had scored 29. When asked how she solved this she said that she made her own number line, ‘like the teacher gets me to do in class’. After observing another pupil solve the problem, using cubes to represent each goal and ‘counting all’, when given the same problem with different numbers, Lara also used cubes to represent each goal and ‘counted all’ to reach the correct answer.

Lara was asked for two numbers that add up to 10. She said 8 + 9. When asked if there were any others and she said 5 + 5 then, ‘that’s all I know’.

Equal sharing problem

Lara was asked to count out 12 cubes and to share them equally with three toys so that each toy got the same number of cubes. Lara gave each toy one cube at a time counting, ‘one, one, one- two, two, two …’ and so on until each toy had four cubes. Without moving the previous cubes, another toy was added and Lara was asked if she should make sure that they all still got the same number of cubes now that this toy had come along. Lara took one cube from each puppet and gave it to the new one. Lara was asked, ‘How did you know to take one from each?’ Lara answered, ‘so they could get the same amount, I thought they should get 3 each’.

Subtraction

A toy was placed on the table with 8 cubes in front of it and Lara was asked:

> Joe has 8 toys, Jack comes and steals 4. How many does Joe have now?

Lara removed 4 cubes, counted the remaining cubes starting from 1 and said '4'.

Lara was then asked, (without setting out the toy and cubes beforehand but having them available):

> Joe has 9 toys and Jack steals 5. How many toys does Joe have left?

Lara took a toy and counted out 9 cubes to put in front of it. She took another toy to remove 5 cubes, one by one. When I asked, 'How many did Joe have left?' Lara replied immediately, '4'.

Missing addend

Lara was given the word problem*:*

> There are 5 children on the bus. Some children get on the bus at the bus stop. Now there are 12 children on the bus. How many children got on at the bus stop?

Lara drew a bus and asked me to repeat the problem. As I repeat it, when I say 5, she draws 5 windows. I continue with the problem and when she hears 12, Lara says 'oh, 12!' and draws more windows, as she is drawing the windows she asks, 'Can you tell me when to stop?' I say, why don't you show me where you think is right. Lara draws 11 windows and draws heads on them. She then tells me, '12 got on at the bus stop'.

Combinations to 5

I asked Lara 'Do you know any numbers that you can put together to make 5?'

> Lara::*'five plus five'*.
>
> Me:: *'Does that make 5?'*
>
> Lara::*'I don't know'*.

I give Lara 5 cubes and tell her she can use these to help her. Lara splits the cubes into 3 groups and says, '1 and 2 and 2'. I ask Lara if she can find another way. She moves the cubes to make 3 + 2 then 4 + 1.

Multiplication

Lara was given the word problem:

> Sean has 3 boxes of toys. There are 6 toys in each box, how many toys does Sean have altogether?

Lara drew 3 boxes and puts 6 dots in each, then counted all starting from one before answering '18'.

Measurement division

Lara was given the word problem:

> Mrs Watson has 24 cookies. She puts them into boxes so that there are 6 cookies in each box. How many boxes of cookies does Mrs Watson have altogether?

Lara drew 2 boxes and put 6 dots in each then stopped. She was reminded that there are 24 cookies altogether and there are 6 cookies in each box; Lara then went back and put more dots until she has 24 cookies in each box before stopping.

Algebraic reasoning

Lara was shown the equation 4 + 3=7 on number cards and asked to change these to make another sum (the term familiar to Lara). Lara changed it to 3 + 4=7. She was then asked to change it into a subtraction sum. 'Lara asks me what subtraction is, I tell her 'take away'. Lara set out 7 − 4 = 3. I show Lara the expression 8 + 6=14 and ask her to make this into another sum, she changes it to 6 − 8 = 14'.

Analysis

The illustrations from the support teacher's records provide an indication of the interactive basis of the assessment process with Lara. It was clear that the support teacher was focussing on the process which Lara was demonstrating and the knowledge gained from this informed immediate as well as more long-term instructional decisions. However, interpreting and responding to a child's solution strategies and using this knowledge as the basis for instructional decisions requires significant knowledge (Empson and Jacobs, 2008; Jacobs *et al*., 2010). This deep knowledge (Ma, 1999) permits clear and focussed in-the-moment instructional decisions, for example, giving Lara a multiplication problem and then a division problem and extending that by asking 'what if there was one more toy?' in the equal sharing problem focusses on the process rather than the product of learning (Munn, 2008).

Understanding the details of children's mathematical thinking is central to supporting them in their learning (Wright *et al*., 2000; Walshaw, 2012). Noticing specific details of how Lara approached particular problems allowed the support teacher to make important pedagogical decisions both in terms of her immediate interaction with Lara and also next steps in planning for Lara in collaboration with the class teacher. For example, observations of Lara's counting informed decisions about number combinations in problems being given to her. Recognising that she was using a 'count all' strategy (Maclellan, 1995) and was directly modelling her solutions (Carpenter *et al*., 1999) was important in considering how to support and extend Lara's mathematical understanding through appropriately designed problems and counting activities. Although Lara had a good forward number sequence, in referring to numbers independently of the sequence she demonstrated difficulty with numbers beyond 20, confusing teens with decades. Lara was also unsure of backward number sequences and had difficulty in understanding the language of 'before' and 'after' in relation to the number sequence but she did demonstrate partial understandings and an ability to count back in tens from 90, albeit out of sequence (Johnson *et al*., 2019).

The support teacher's observations also indicated that prior explicit instruction could get in the way of a child's understanding. This has been found to be problematic particularly for children who struggle with mathematics (Behrend, 2003). However, when children are encouraged to adopt a sense-making approach the evidence is that they gain greater conceptual as well as procedural understanding (Carpenter *et al*., 2000; Fosnot and Dolk, 2001; Baroody, 2006; Moscardini, 2009). For example, Lara wanted to attempt the 2 + 9 addition word problem which had been given to another child in the class, she chose not to use the tools available instead she attempted to follow the taught strategy explicitly shown by the teacher. This resulted in her taking the numbers from either end of her self-created number line and combining these to create her answer indicating her lack of conceptual understanding (Askew, 2010). However, when she saw another child modelling the problem with materials, Lara also chose to model to make sense of the problem which allowed her to build on the strategy that she used to solve the problem in the unscreened addition task. It also illustrates the social aspect of dynamic assessment highlighted by Hill (2015, p. 130).

The observations indicate that Lara used direct modelling strategies and attended to the structure of the problem by following the language of the problem (Carpenter *et al*., 2000). By recognising this, the support teacher presented Lara with multiplication and division problems (of which Lara had had no prior experience) with suitable number combinations which were aligned to the observations of her counting. The decision to introduce problems of multiplication and division was a dynamic response informed by observations of Lara's ability to direct model rather than as part of a pre-determined instructional sequence set out in an external curriculum framework (Coffield and Edward, 2009). Lara used a direct modelling strategy to solve a multiplication problem by drawing out the problem (3×6). The support teacher recognised that Lara was beginning to develop abstract thinking by representing parts of the problem as marks on the whiteboard.

When attempting a problem of division, Lara was able to share out cubes between toys by acting out the problem she was given, however she required the correct number of cubes to be already available and the correct number of toys that the cubes were being shared between in order to do this. She was unable to do this when solving a problem when she was not given the exact number of cubes and she became confused when trying to represent objects in the more abstract form of a picture. This may show that Lara still requires further experience of solving division problems in a clear real-life context that is available to her before moving onto early level abstract contexts (Baroody, 2006).

Lara had demonstrated an ability to solve the multiplication and addition problems as well as result-unknown problems. The support teacher considered if she could extend this by giving Lara a problem involving a missing addend. However, this is a challenging problem. When solving the problem Lara recognised some parts of the problem that she was able to model out however she was unsure of what to do with these. Rather than explicitly teaching strategies that might not be meaningful the support teacher moved to mathematical problems that she could directly model at a basic level and which provided important information for planning next steps for Lara.

When shown four cubes and asked how many had been taken away, Lara was able to give the correct answer. However, when taking the same number of cubes away from five, Lara gave an incorrect response. This could possibly be down to Lara's ability to subitise, as children learn to subitise small numbers, usually up to three or four (Mix *et al*., 2012). Lara may have been able to subitise numbers up to four and so was able to say how many were taken away from that number, but she could not subitise five. Also, as she did not yet have a relational understanding of five and the numbers within it this made the task challenging. This was evidenced further when Lara was unable to give two numbers which make five when she was asked, showing she does not know these by memory. However, when given five cubes and asked to use them to create different ways of making five, Lara was able to do this, showing that she could use a modelling strategy to solve this, something which can be built on to develop her understanding of number facts further. Carpenter *et al*. (2000) found evidence that children taught using word problems can actually have better recall of number facts than those who are not. This supports the view that if children have difficulty recalling number facts, building understanding through their engagement with word problems helps them to make connections between numbers and consequently supports the development of number facts.

Lara was only able to give one number bond for 10, despite this having been a focus of classroom planning for several months before the observations took place. As illustrated in the algebraic reasoning task, her lack of any relational understanding of number bonds and how number facts were connected to one another, hampered her ability to draw upon and utilise that knowledge (Askew, 2010).

Although there was some recognition of Lara's difficulties this did not lead to any specific response or individualised programme, Lara continued to follow the general curricular planning of the class. The significance of the findings is in the detailed picture of Lara's mathematical understanding which emerged and how this could be used to inform instruction. The development of this detailed picture was dependent on the teacher's knowledge of children's mathematical thinking. In the case study it was the support teacher who had this deep knowledge base. The following discussion will consider the significance of this specifically in relation to the process of dynamic assessment. While it is recognised that the educational support model in practice involved collaboration between two teachers, it is beyond the scope of this paper to explore this collaboration in depth but it is recognised as an area for further investigation.

Discussion

The case study provides further evidence of the capacity of a teacher with deep knowledge to assess and support the learning of a child demonstrating difficulties in the area of mathematics. While no claims are being made as far generalisability is concerned, the richness of the case provides an opportunity to reflect on the relationship between a teacher's knowledge of the mathematical thinking of a particular child and engaging in a process of dynamic assessment.

Making an explicit link between teachers' knowledge of children's mathematical thinking and the application of this knowledge within the context of the process dynamic assessment supports the development of inclusive practice. Knowledge of individual children and particularly how they are making sense of their learning affords the classroom teacher the opportunity to identify and support a child in a way that is commensurate with inclusive practice (Rogoff *et al*., 1996; Watson, 1996; 2001; Lewis and Norwich; 2000; Dyson, 2001). This is not a new idea. Such practice is not in denial of the need to respond to an individual or at the level of the individual but it rejects more traditional and essentially remedial practices which have long been the hallmark of special education (Tomlinson, 1982; Thomas and Loxley, 2007; Thomas, 2013; Rix, 2015). However, there is evidence of a resurgence of in-child deficit models in initial teacher education (for example, Ellis, 2017), which, in relation to mathematics education fail to take into account issues of pedagogy and children's mathematical thinking. The current political priority in the UK to raise standards in mathematics, specifically by closing the gap between low-attaining and higher attaining learners, requires consideration of the underlying causes of low-attainment both structurally and pedagogically (Dyson and Hick, 2005; Hallam and Parsons, 2013; Tomlinson, 1982). The significance of connecting teachers' knowledge of children's mathematical thinking to a process of dynamic assessment is that it positions teachers to attend to particular children within the context of their regular practice and in so doing promotes a more inclusive form of practice and encourages a shift away from responses that reflect a within-child deficit or medical model.

By making an explicit connection between teachers' attending and responding to children's mathematical thinking and dynamic assessment, it makes clear that the teacher plays a pivotal role in identifying and consequently supporting any children in need of support in their mathematical learning. Action is immediate and built into the process; this can be based on a holistic picture informed by knowledge of the child's mathematical thinking along with an understanding of the child's contextual and cultural background. Jacob's *et al*. (2010) set out an influential framework for responding to children's mathematical thinking. Central to this framework is the idea of teachers interpreting children's solution strategies and using these interpretations to inform instruction. Russ (2017) emphasises the importance of teachers engaging with students' sense-making and makes an important distinction between teachers' noticing at a descriptive level from an interpretive level. Jacob *et al*.'s (2010) framework outlines the kind of detailed information which an informed teacher might realistically be expected to produce about an individual learner. It is this detail which is firmly in the domain of mathematics learning and teaching and arguably reflects the quality of interpretation of a child's thinking called for by Russ (2017). As far as the current study is concerned, the support teacher had significant experience as a CGI practitioner and while it cannot be assumed that all teachers will achieve the same degree of competency there is evidence that teachers are responsive to high quality professional development in CGI (Fennema *et al*., 1996; Franke *et al*., 2001; Steinberg *et al*., 2004; Moscardini, 2015).

The potential to develop more inclusive practice through aligning dynamic assessment with teaching and specifically with classroom-based intervention has been recognised and called for by some educational psychologists (see Elliot, 2003; Stringer, 2009). Stringer outlines three elements central to Feuerstein's model: a recognition of the dynamic of intelligence, the importance of mediation and a cognitive map for analysis. This is consistent with the features outlined by Grigorenko and Sternberg (1998) namely, a recognition of the psychological process involved in learning; the provision of feedback in a way that connects the assessment process with instruction and recognition of the interactive relationship between the student and the assessor (Grigorenko and Sternberg, 1998, p. 75). Each of these elements was evident in practice in the case study; most importantly, the support teacher had the requisite knowledge of children's mathematical thinking to permit an informed interpretation.

In the case study, understanding Lara's mathematical thinking allowed the support teacher to collaborate with the class teacher in developing an informed response. The circumstances prior to this were that while there was some concern about Lara struggling in maths, this did not result in any action. Arguably this is not an untypical scenario – a child's difficulty may go unnoticed, perhaps because there is no mechanism for identification other than cumulative failure until such time that the gap between the child's understanding and the mathematics being taught in the classroom is glaring. At this point, remedial action may be taken through various forms of differentiation, possibly referral for support from out with the classroom and potentially a diagnosis of a specific condition, for example dyscalculia. This reflects a traditional model in which children who are seen as failing are ‘moved to the resources’ (Dyson, 1991). Concern about lack of progress is seen as a pupil deficit with little, if any, consideration given to pedagogy and the learning environment as a barrier to learning (Mara and Mara, 2012). Farrell and Venables (2009) consider this issue from the perspective of educational psychologists and argue for a shift away from a referral process which focuses on the child as the person with the problem towards a collaborative approach which recognises barriers to learning and participation as external to the child.

It is within the context of this collaborative model which an informed teacher has much to offer. While an educational psychologist is undoubtedly well-placed to administer an individual assessment of a child, and Farrell and Venables describe the referral of a child who has difficulties in learning as the most common type of referral to an educational psychologist (p. 120), this assessment may focus on what the child is not capable of doing and provides an explanation of the problem as located within the child. Furthermore, the recommendations from the educational psychologist about next steps for the child in numeracy may be limited to the knowledge base of the educational psychologist in that area with an expectation that the details of this are in the domain of the class teacher. A similar argument has been presented by psychologists in the field of literacy (Elliott and Gibbs, 2008).

In the case study, the support teacher’s detailed and informed observations allowed her to make appropriate and specific recommendations for the pupil. It was evident that Lara did not yet have the relational understanding or the awareness of place value that would enable her to make sense of the vertical addition of double digit numbers currently being taught to the whole class. More importantly, however, the interactive nature of the assessment process helped to provide a clearer sense of Lara’s understanding and how this might inform subsequent teaching. Munn described this interactive basis as central to the process of dynamic assessment in which the assessor does not try to be objective but rather ‘explores the learner’s thinking through reciprocal interaction’ (Munn, 2008). This is a dynamic response to information about learners based on their actual responses to real interventions (Lidz, 2014 p. 295). In so doing, it takes assessment beyond screening and connects it with the teaching process, a process which is informed by knowledge of children’s thinking (Empson and Jacobs, 2008; Resing *et al.*, 2016). Responding dynamically in the way described supports a more inclusive practice in which the assessment process connects with teaching rather than being used for 'educational selection' (Elliot, 2003, p. 18). It also represents a more fluid and responsive approach to educational support which has the potential to be embedded in classroom practice (Norwich and Ylonen, 2015).

Conclusion

Viewing dynamic assessment from an educational perspective and specifically recognising the potential capacity of the teacher to build a cognitive map of the learner support the fulfilment of responding to a child's needs dynamically as opposed to recognising and responding to failure. This is in line with current educational support procedures, exemplified for example, by the *graduated approach* adopted in England and *Staged Intervention* as practised in Scotland. These models require the regular class teacher to be the initial source of information gathering about a child. The interaction between assessment and teaching and how this comes to be integrated promotes a more inclusive response and a further shift away from a discrepancy model with the attendant risk that some children may be deemed to require specialised support when this may not be necessary while others may be overlooked and not receive the support which they require (Grigorenko, 2009, p. 115). Assessment needs to move beyond screening and provide valuable information that informs teaching (Lidz, 2014). Recognising the fundamental role of the teacher, Penny Munn summed up this process neatly,

> 'What teachers need to be able to do is simply recognise the 'cutting edges' of a child or group of children's maths development and then teach around that edge letting the child take more and more control of the direction of learning' (Munn, 2008).

It is as hard and as simple as that. The challenge then is one of how to best support teachers in the development of this dynamic practice.

Acknowledgement

The authors dedicate this paper to the memory of Dr Penny Munn, University of Strathclyde, who made a lasting contribution to our understanding of children's thinking.

References

ASKEW, M. (2010) It ain't (just) what you do: effective teachers of numeracy. In I. Thompson (ed.), *Issues in Teaching Numeracy in Primary Schools*. 2, Berkshire: Open University Press. 31–44.

BALL, D. L., THAMES, M. H. and PHELPS, G. (2008) Content knowledge for teaching: what makes it special? *Journal of Teacher Education*, 59, 5, 389–407. doi:https://doi.org/10.1177/0022487108324554.

BAROODY, A. J. (2006) Why children have difficulties mastering the basic number combinations and how to help them. *Teaching Children Mathematics*, 13, 22–31.

BEHREND, J. L. (2003) Learning-disabled students make sense of mathematics. *Teaching Children Mathematics*, 9, 5, 269–274.

CARPENTER, T. P., FENNEMA, E., FRANKE, M. L., LEVI, L. and EMPSON, S. B. (1999) *Children's Mathematics: Cognitively Guided Instruction*. Portsmouth, NH: Heinemann.

CARPENTER, T. P., FENNEMA, E., LOEF FRANKE, M., LEVI, L. and EMPSON, S. (2000) *Cognitively Guided Instruction: A Research-Based Teacher Professional Development Program for Elementary School Mathematics. Research Report*. Madison: NCISLA.

COFFIELD, F. and EDWARD, S. (2009) Rolling out 'good', 'best' and 'excellent' Practice. What next? Perfect practice? *British Educational Research Journal*, 35, 3, 371–390. doi:https://doi.org/10.1080/01411920802044396.

DYSON, A. (1991) Rethinking roles, rethinking concepts: special needs teachers in mainstream schools. *Support for Learning*, 6, 2, 51–60. doi:https://doi.org/10.1111/j.1467-9604.1991.tb00410.x.

DYSON, A. (2001) Special needs education as the way to equity: an alternative approach? *Support for Learning*, 16(3), 99–104.

DYSON, A. and HICK, P. (2005) Low attainment. In A. Lewis and B. Norwich (eds), *Special Teaching for Special Children? Pedagogies for Inclusion*. Berkshire: OUP. 192–205.

ELLIOT, J. (2003) Dynamic assessment in educational settings: realising potential. *Educational Review*, 55, 1, 15–32. doi:https://doi.org/10.1080/00131910303253.

ELLIOT, J. G. (2000) Dynamic assessment in educational contexts: purpose and promise. In C. S. Lidz and J. G. Elliot (eds), *Dynamic Assessment: Prevailing Models and Applications*. New York: Elsevier. 713–740.

ELLIOTT, J. G. and GIBBS, S. (2008) Does dyslexia exist? *Journal of Philosophy of Education*, 42, 3–4, 475–491. doi:https://doi.org/10.1111/j.1467-9752.2008.00653.x.

ELLIOT, J., LAUCHLAN, F. and STRINGER, P. (1996) Dynamic assessment and its potential for educational psychologists. Part 1: theory and practice. *Educational Psychology in Practice*, 12, 3, 152–160. doi:https://doi.org/10.1080/0266736960120303.

ELLIOTT, J. and RESING, R. (2015) Can intelligence testing inform educational intervention for children with reading disability? *Journal of Intelligence*, 3, 4, 137–157. doi:https://doi.org/10.3390/jintelligence3040137.

ELLIS, S. (2017) The Strathclyde literacy clinic: developing student teacher values, knowledge and identity as inclusive practitioners. In M. Peters, B. Cowie and I. Menter (eds), *A Companion to Research in Teacher Education*. Singapore: Springer. 121–133.

EMPSON, S. and JACOBS, V. R. (2008) Learning to listen to children's mathematics. In: D. Tirosh and T. Woods (eds), *Tools and Processes in Mathematics Teacher Education*. Rotterdam: Sense Publishers. 257–281.

FARRELL, P. T. and VENABLES, K. (2009) Can educational psychologits be inclusive? In P. Hick, R. Kershner and P. T. Farrell (eds), *Psychology for Inclusive Education: New Directions in Theory and Practice*. Oxon: Routledge. 117–126.

FENNEMA, E., CARPENTER, T. P., FRANKE, M. L., LEVI, L., JACOBS, V. R. and EMPSON, S. B. (1996) A longitudinal study of learning to use children's thinking in mathematics instruction. *Journal for Research in Mathematics Education*, 27, 4), 403–434. doi:https://doi.org/10.2307/749875.

FOSNOT, C. and DOLK, M. (2001) *Young Mathematicians at Work: Constructing Number Sense, Addition and Subtraction*. Portsmouth: Heinemann.

FRANKE, M. L., CARPENTER, T. P., LEVI, L. and FENNEMA, E. (2001) Capturing teachers' generative change: a follow-up study of professional development in mathematics. *American Educational Research Journal*, 38, 3, 653–689. doi:https://doi.org/10.3102/00028312038003653.

FRANKE, M. L. and KAZEMI, E. (2001) Learning to teach mathematics: focus on student thinking. *Theory into Practice*, 40, 2, 102–109.

GOEPEL, J. (2009) Constructing the individual education plan: confusion or collaboration? *Support for Learning*, 24, 3, 126–132. doi:https://doi.org/10.1111/j.1467-9604.2009.01412.x.

GRIGORENKO, E. L. (2009) Dynamic assessment and response to intervention: two sides of one coin. *Journal of Learning Disabilities*, 42, 2, 111–132.

GRIGORENKO, E. L. and STERNBERG, R. J. (1998) Dynamic testing. *Psychological Bulletin*, 124, 1, 75–111.

HALLAM, S. and PARSONS, S. (2013) The incidence and make up of ability grouped sets in the UK primary school. *Research Papers in Education*, 28, 4, 393–420. doi:https://doi.org/10.1080/02671522.2012.729079.

HILL, J. (2015) How useful is dynamic assessment as an approach to service delivery within educational psychology? *Educational Psychology in Practice*, 31, 2, 127–136. doi:https://doi.org/10.1080/02667363.2014.994737.

HILL, H., BALL, D. H. and SCHILLING, S. (2008) Unpacking "pedagogical content knowledge": conceptualizing and measuring teachers' topic-specific knowledge of students. *Journal for Research in Mathematics Education*, 39, 4, 372–400.

HUNT, J. H. and EMPSON, S. B. (2014) Exploratory study of informal strategies for equal sharing problems of students with learning disabilities. *Learning Disability Quarterly*, 38, 4, 208–220. doi:https://doi.org/10.1177/0731948714551418.

JACOBS, V. R., LAMB, L. L. C. and PHILIPP, R. A. (2010) Professional noticing of children's mathematical thinking. *Journal for Research in Mathematics Education*, 41, 169–202.

JOHNSON, N. C., TURROU, A. C., MCMILLAN, B. G., RAYGOZA, M. C. and FRANKE, M. L. (2019) Can you help me count these pennies? Surfacing preschoolers' understandings of counting. *Mathematical Thinking and Learning*, 21, 4, 237–264. doi:https://doi.org/10.1080/10986065.2019.1588206.

LEWIS, A. and NORWICH, B. (2005) *In Special Teaching for Special Children? Pedagogies for Inclusion*. Berkshire: Open University Press.

LEWIS, A. and NORWICH, B. (2000). *Mapping a Pedagogy for Learning Difficulties*. Report submitted to the British Educational Research Association (BERA), February 2000.

LIDZ, C. S. (2014) Leaning toward a consensus about dynamic assessment: can we? Do we want to? *Journal of Cognitive Education and Psychology*, 13, 3, 292–307. doi:https://doi.org/10.1891/1945-8959.13.3.292.

LIDZ, C. S. (1995) Dynamic assessment and the legacy of L.S. Vygotsky. *School Psychology International*, 16, 143–153.

Lidz, C. S. and Elliot, J. G. (eds) (2000) *Dynamic Assessment: Prevailing Models and Applications*. New York: Elsevier.

MA, L. (1999) *Knowing and Teaching Elementary Mathematics: Teachers' Understanding of Fundamental Mathematics in China and the United State*. Mahwah, NJ: Lawrence Erlbaum Associates.

MACLELLAN, E. (1995) Counting all, counting on, counting up, counting down. *Education 3-13*, 23, 3, 17–21.

MARA, D. and MARA, E. L. (2012) Curriculum adaption in inclusive education. *Procedia - Social and Behavioral Sciences*, 46, 4004–4009.

MERRIAM, S. B. (1998) *Qualitative Research and Case Study Application in Education*. San Francisco, CA: Jossey-Bass.

MIX, K. S., SANDHOFER, C. M., MOORE, J. A. and RUSSELL, C. (2012) Acquisition of the cardinal word principle: the role of input. *Early Childhood Research Quarterly*, 27, 274–283.

MOSCARDINI, L. (2015) Primary special school teachers' knowledge and beliefs about supporting learning in numeracy. *Journal of Research in Special Educational Needs*, 15, 1, 37–47. doi:https://doi.org/10.1111/1471-3802.12042.

MOSCARDINI, L. (2014) Developing equitable elementary mathematics classrooms through teachers learning about children's mathematical thinking: cognitively guided instruction as an inclusive pedagogy. *Teaching and Teacher Education*, 43, 69–79.

MOSCARDINI, L. (2010) 'I like it instead of maths': how pupils with moderate learning difficulties in Scottish special schools intuitively solved mathematical word problems. *British Journal of Special Education*, 37, 3, 130–138.

MOSCARDINI, L. (2009) Tools or crutches? Apparatus as a sense-making aid in mathematics teaching with children with moderate learning difficulties. *Support for Learning*, 24, 1, 35–41.

MUNN, P. (2008) *Dynamic Assessment: Linking Assessment and Teaching Frameworks*. Keynote Lecture for US Math Recovery Conference, Ohio, April 16th 2008

NORWICH, B. and YLONEN, A. (2015) A design-based trial of lesson study for assessment purposes: evaluating a new classroom based dynamic assessment approach. *European Journal of Special Needs Education*, 30, 2, 253–273. doi:https://doi.org/10.1080/08856257.2015.1009702.

RESING, W. C. M., BAKKER, M., PRONK, C. M. E. and ELLIOTT, J. G. (2016) Dynamic testing and transfer: an examination of children's problem-solving strategies. *Learning and Individual Differences*, 49, 110–119. doi:https://doi.org/10.1016/j.lindif.2016.05.011.

RIX, J. (2015) *Must Inclusion be Special? Rethinking Educational Support Within a Community of Provision*. London: Routledge.

ROGOFF, B., MATUSOV, E. and WHITE, C. (1996) Models of teaching and learning: participation in a community of learners. In D. R. Olson and N. Torrance (eds), *The Handbook of Education and Human Development*. Oxford: Blackwell. 318–414.

RUSS, R. S. (2017) Characterizing teacher attention to student thinking: a role for epistemological messages. *Journal of Research in Science Teaching*, 55, 1, 94–120. doi:https://doi.org/10.1002/tea.21414.

SANCHES-FERREIRA, MANUELA, LOPES-DOS-SANTOS, PEDRO and ALVES, SÍLVIA (2013) How individualised are the individualised education programmes (IEPs): an analysis of the contents and quality of the IEPs goals. *European Journal of Special Needs*, 28, 4, 507–520. doi:https://doi.org/10.1080/08856257.2013.830435.

SHULMAN, L. S. (1986) Those who understand: knowledge growth in teaching. *Educational Researcher*, 15, 2, 4–14.
STAKE, R. E. (1995) *The Art of Case Study Research*. London: Sage.
STEINBERG, R. M., EMPSON, S. B. and CARPENTER, T. P. (2004) Inquiry into children's mathematical thinking as a means to teacher change. *Journal of Mathematics Teacher Education*, 7, 237–267.
STRINGER, P. (2009) Dynamic assessment for inclusive learning. In P. Hick, R. Kershner, & P. T. Farrell (eds), *Psychology for Inclusive Education: New Directions in Theory and Practice*. Oxon: Routledge. 127–138.
STRINGER, P., ELLIOT, J. and LAUCHLAN, F. (1997) Dynamic assessment and its potential for educational psychologists: part 2 - the zone of next development. *Educational Psychology in Practice*, 12(4), 234–239. doi:https://doi.org/10.1080/0266736970120409.
THAMES, M. H. and BALL, D. L. (2010) What math knowledge does teaching require? *Teaching Children Mathematics*, 17, 4, 220–229.
THOMAS, G. (2013) A review of thinking and research about inclusive education policy, with suggestions for a new kind of inclusive thinking. *British Educational Research Journal*, 39, 3, 473–490.
THOMAS, G. and LOXLEY, A. (2007) *Deconstructing Special Education and Constructing Inclusion*. Berkshire: Open University Press.
TOMLINSON, S. (1982) *A Sociology of Special*. London: Routledge and Kegan Paul.
TORRANCE, H. and PRYOR, J. (1998) *Investigating Formative Assessment: Teaching, Learning and Assessment in the Classroom*. Buckingham: OUP.
TURNER, E. E. and DRAKE, C. (2016) A review of research on prospective teachers' learning about children's mathematical thinking and cultural funds of knowledge. *Journal of Teacher Education*, 67, 1, 32–46. doi:https://doi.org/10.1177/0022487115597476.
WALSHAW, M. (2012) Teacher knowledge as fundamental to effective teaching practice. *Journal of Mathematics Teacher Education*, 15, 3, 181–185. doi:https://doi.org/10.1007/s10857-012-9217-0.
WATSON, J. (1996) *Reflection through interaction: The classroom experience of pupils with learning difficulties*. London: Falmer,
WATSON, J. (2001) Social constructivism in the classroom. *Support for Learning*, 16, 3, 140–148.
WRIGHT, R. J., MARTLAND, J. and STAFFORD, A. (2000) *Early Numeracy: Assessment for Teaching and Intervention*. London: Paul Chapman Publications/Sage.
YIN, R. K. (2003) *Case Study Research- Design and Methods* (third edition). Thousand Oaks: Sage.

Correspondence
Lio Moscardini
Royal Conservatoire of Scotland 100 Renfrew St, Glasgow G2 3DB
Email: lmoscardini@rcs.ac.uk

