# Visualizing and Investigating Big Ideas Mathematics GRADE 3



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## Mindset Mathematics Visualizing and Investigating Big Ideas

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#### **Data Tells Us about Ourselves** Snapshot

What is the most common car color where you live? Students develop and try out a data collection plan to answer this question, and then look across the data collected by the whole class to see what can be learned from the similarities and differences. We offer two options



for multiday extensions that put students in charge of using data to investigate their school or community.

Connection to CCSS 3.MD.3

#### Agenda

Activity	Time	Description/Prompt	Materials
Launch	10–15 min	Show students the Parking Lot Photo and invite observations. Ask students to predict, What is the most com- mon car color in our area? As a class, develop a data collection plan that would help answer this question.	Parking Lot Photo, to display
Explore	30+ min	Partners work together to imple- ment the class's data collection plan. Students develop ways to organize and display the data. Partners make observations and use their evidence to come to a conclusion, which they include on a display.	<ul> <li>Charts and markers</li> <li>Data collection and display tools, such as clipboards, grid pa- per (see appendix), and tape</li> </ul>
Discuss	20 min	Do a gallery walk of all the data and conclusions from all partnerships. Dis- cuss the similarities and differences in the data the groups collected, and what may have led to differences. Come to a class conclusion about the most common car color in your area.	

(Continued)

Acti	vity	Time	Description/Prompt	Materials
Exte	end	Varies: 2+ days	Two choices are offered for exten- sion: investigation of a classroom- or school-based question, or the devel- opment of a community-based re- search project. Both require defining a question, developing a data collection plan, collecting and displaying data, and coming to a conclusion to share.	Data collection and display tools, such as clipboards, grid paper (see appendix), tape, charts, and markers

#### To the Teacher

In this investigation, we have approached developing the skills to use data to answer questions with a question we think any class could reasonably ask and answer with locally gathered data: What is the most common car color in your area? We aim to support students to develop a plan for collecting and examining data that helps them address the question. We encourage you to think about the ways that the physical location and resources of your school site might give students opportunities to collect data. Can you see cars driving by your classroom window? Do they go by often enough to get a real sample in a reasonable time? Does your school have a large (and, we hope, calm) parking lot? Is there parallel parking on the street in front of your school? Could you take students on a walking tour of your neighborhood to tally parked cars? We have not dictated a plan in this lesson because students should develop the plan, and it must be reasonable in your context. Thinking through the logistics of data collection in advance will help you respond to students' ideas and take up those that are safe and reasonable.

Students do not need to all use the same plan or collect data simultaneously. You might decide to break the class into smaller groups to do data collection in shifts. The differences in the data the groups collect will only add to the later whole-class discussion. Partners should be in charge of deciding how much data they need, and, to the extent that is practical, we encourage you to give students choices about how to collect the data. For instance, some students may want to just collect from one full row of the parking lot, while others want to do the back half, and still others want to try to count them all. These different choices will give your class lots to talk about in the end.

We think this initial investigation will give students many opportunities to learn about using data from the real world to answer questions, but we recognize the limitations of the specific question we've posed. Students will be far more motivated to collect and analyze data if they are attempting to answer their own questions. In the extension of this lesson, we've offered two different ideas for multiday investigations of authentic questions. Students will build on what they learn investigating the car question to pose and research their own questions. These extensions are open ended and take some planning to tailor them to the opportunities in your school and community. We think it's worth the work. Students often have data thrust at them, or gain experience with data conducting surveys of things they aren't really invested in. We think choosing one of the extension options here will give students avenues for seeing the power of data in the real world.

#### Activity

#### Launch

Launch by showing students the Parking Lot Photo and asking them what they notice. Give students a chance to turn and talk, and then take students' observations. Note that students may notice things about how the cars are arranged and the colors. Probe students to provide some evidence for their observations, particularly observations that compare (such as, "There are more white cars than orange.") or estimate (such as, "I think there are more than 100 cars there.").



Source: Image by Shutterstock.com/Aleksei Kazachok.

Pose the question, What is the most common car color in our area? Ask students to make a prediction based on the photo or what they know. Ask students to share their predictions and their reasoning. Be sure to make connections between their evidence and their prediction. For instance, students might say that they predict white is the most common because of all the white cars in the photo. You would want to make the connection clear that they are only seeing some cars, but they think that this photo represents all cars. You might ask them if they think this photo would match the cars where you live.

Ask, How could we collect data to answer this question? Tell students that you don't have a photo like this one taken where you live, so you will need a different way to collect data to answer the question. Possibilities include counting cars in the parking lot, cars parallel parked on the street in front of the school, or cars as they drive by. Work together to come up with either one shared plan for the class or a couple of possibilities that you can reasonably support.

#### **Explore**

Students work in partners to collect data following the class plan or choosing from the plans the class developed. Students need to figure out a way to record the data they collect, organizing it so they can use it to make conclusions. You may want to take students out of the building to do a walking survey of cars in a particular area, or students may be able to observe cars through the classroom window or from another indoor vantage point.

Once partners have collected what they think is enough data, they need to design a way to display the data to help answer the question, What is the most common car color in our area? Students should be encouraged to display all of their data, even the data for colors that are less popular, so that the class can make comparisons later. Their displays should include all data, their conclusion about the most popular color, and how their data supports that conclusion.

#### Discuss

Have students post their displays, including any graph and table data they have, their conclusions, and their supporting evidence. Do a gallery walk, in which students walk around and look at the different displays and conclusions. As they walk, student should be thinking about the following questions:

- How is our data similar?
- How is our data different? Why might our data be different?
- What appears to be the most common color in our area?

Discuss the differences and similarities in the data collected by the different groups, and ideas about why the data might not be the same. Ask students:

- What conclusions can we draw about the most common color in our area?
- What data did you find most convincing? Why?
- What other data would we want to collect to be sure?

#### Extend

There are two possibilities for extension, depending on what would excite your students:

- 1. Ask students to pose their own question about their school or community that they could reasonably collect data to answer. Note that students are often encouraged to ask questions that focus on favorites, but in this investigation, we'd like to go beyond making conclusions about what the students in one class like, toward being able to think about the school, neighborhood, or community. Work with students to generate questions that intrigue them and where data could be collected, such as:
  - How much waste is created by each grade level (or class or lunch period) at lunch?
  - Which lunches do the teachers eat? Which lunches do the kids eat?
  - How do kids get to school?
  - What do kids do after school? How does that change each day?
  - How much paper does each class recycle?
  - How much litter is left in each hallway (or in front of each classroom) each day?
  - What books are checked out each week from the library?

Note that these are just examples, and we encourage you to brainstorm as a class. Students should make a prediction (or hypothesis) about the question and then devise a way to collect the needed data. Students will then need to create a display and a way to share. Students should make observations about their data that help answer their question or notice surprising findings. Create time for each group to share what they have found and for the class to ask questions.

- 2. Identify an area in your school, classroom, or community that genuinely needs research for decision making. This could be something like designing a new playground, determining what new books to order for the classroom or school library, planning for a school or classroom event (such as back-to-school night or a party), or making a recommendation to the community or town on a new project (such as putting together a softball league, planning for a community garden, or dealing with a community space that is not well cared for or used). Here the extension is more involved and more authentically similar to the work researchers do. Students need to:
  - Define the question(*s*) that need to be explored.
  - Determine how to gather data to answer those questions. Whom can we ask? Where can we find the data?
  - Collect data and determine how to represent the data so that observations can be made.
  - Determine what the data does and does not tell about the questions that were asked.
  - Make a recommendation based on the data to support the decision making.
  - Present their findings to an audience.

Although planning for and executing this extension could take considerable time, we think students would be deeply engaged in supporting an authentic decision-making process with data and recommendations.

#### Look-Fors

• How are students deciding what cars to sample? Sampling is not a concept that we typically discuss with students in elementary school, but the question we have asked about common car color is one where sampling matters. You'll likely only have access to a sample near your school, of cars either parked nearby or driving past, but students do have a choice of how many cars to sample before they decide they have enough data. Certainly, 5 or 6 cars isn't enough, and a thousand is not necessary. So, how many cars are enough for you to feel confident in your conclusion? This is a great question to be discussing with students as they collect data, and it will create useful variation among

the different groups' data to discuss. The sample size is one factor that will contribute to differences among the data sets, and possibly the conclusions.

- What categories are students using for classifying their data? Students who use large categories, such as *blue*, will get different results from those who use finer categories, such as *light blue*, *navy*, and *teal*. There is no single correct way to create the categories, but consistency is valuable. If students use small categories for some of their data, they should use small categories across their data, where possible. You may want to ask students how they are deciding on the categories and whether they agree on what each category means. There are always marginal cases that need to be resolved, and partners should agree on how to categorize each car.
- Do students' data displays match and make sense? As we worked on in the previous activities in this big idea, different representations of the same data set should match and make sense. It would be surprising for students all to enter the same parking lot and come out with wildly conflicting data. Further, after students tally up the cars, there are lots of opportunities for inadvertent errors to creep in, from counting to recording to displaying. Ask students how they are making sure their data is accurate. Draw students' attention to any discrepancies and help them reason through how these crept in and where the accurate data most likely is.
- How are students supporting their conclusions with evidence? Regardless of the questions students are investigating, they need to use the data reasonably to support conclusions. Students should pick out useful parts of the data to help them address the questions asked. They also need to recognize when multiple answers are reasonable. For instance, if your car survey ends with 42 white cars and 39 blue cars, it would be difficult to claim confidently that white is the most popular, because the two values are too close to be sure. If you notice that your data yields two close leading values, draw the class's attention to what this data means, with particular emphasis on what it *could* mean. In this case, it *could* mean that blue is just as popular as white. This is where we return to the importance of the sample; this is not an election where every vote gets counted and even one vote can create a winner. Here we are getting a small slice of the real data, just a glimpse, and our conclusions need to acknowledge this. If you really want to know which is the most popular, it may mean you need to collect more data. If it is reasonable, encourage students to

do just that. If, in the end, the data shows that it is too close to call, this is a conclusion that can be defended with the data.

#### Reflect

What kinds of questions can data help us answer? How do you decide what data to collect to answer your questions?

#### **Parking Lot Photo**



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