Curved Shapes

Introduction

In Steve Strogatz' Infinite Powers, the reader for our calculus course, Strogatz describes Archimedes search for a way to calculate the area of the curved region between a parabola and a line. Archimedes came up with the strategy of reimagining the parabolic segment as a collection of infinitely many triangular shards glued together. Strogatz reflects that it was through his "noodling around" (2019, p42) that he developed intuition and that it was "an honest account of what it's like to do creative mathematics" (2019, p42) Mathematicians typically develop intuitive ideas before a formal proof, but we rarely ask students in K-16 mathematics education to use their intuition or to think creatively about mathematics – these important acts are devalued or completely absent. In an important experimental study Schwartz and Bransford (1998) found that students were more interested, engaged and academically successful when asked to use their intuition about a solution before being taught formal methods, this made the methods meaningful and gave students an intellectual purpose for learning them. This activity is an occasion for creative and intuitive thinking about the areas of curves, that could be the opportunity for the learning of Reimann Sums and definite integrals.

Materials

- Cutouts of curved shapes (1 shape per group of 2-4 students)
  - ◊ We had a variety of curved shapes for students to choose from, but this activity could be done with just one variation distributed to all groups
  - ◊ Shape cutouts can be made from standard paper however, card stock or foam core boards are preferable becuase they are sturdy
- Whiteboards or poster paper and markers so students can display their work

A few examples of curved shape cutouts

Agenda

Time	Task
10 mins	Launch • Distribute curves • Ask student groups to find the area of their curve with whatever strategies make the most sense to them.

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50 min	Group work • Circulate as groups work and encourage groups who have completed one method to find additional methods for finding the area of their shape • Ask students to prepare a presentation that describes their methods
20 mins	Small group sharing • Students should pair with another group nearby that worked on a different curve. • Each group shares their method for finding the area of the curve (10 mins) • Groups then compare methods and discuss: ◊ How do the methods compare? ◊ What similarities and differences do you notice? ◊ Generalize: Is there a method or aspect of the methods that could work for both shapes
30 mins	Whole group debrief • Have groups share out about the methods they used and the results of their discussions • Is there a method that could work for all shapes? • Possibly tease out vocabulary such as: Riemann sums and integration

References:

Strogatz, Steven. Infinite Powers: How Calculus Reveals the Secrets of the Universe. Houghton Mifflin Harcourt, 2019.

Schwartz, D., & Bransford, J. (1998). A Time for Telling. Cognition and Instruction, 16(4), 475-522.