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What would the nautilus say?

Grades 6-12+

Introduction

While the nautilus shell is often represented in popular culture as an example of a golden spiral, according to many mathematicians it is not. They acknowledge that it is a classic example of a logarithmic spiral, but claim it does not have the growth factor φ (1.618...) required to make it the special case of the logarithmic spiral traditionally deemed the golden spiral. We ask, could an alternative frame or change in axioms give the nautilus shell a golden hue? What other patterns might be hiding in it's shell?

We offer two different task pages so you can choose what is appropriate for your students. We are excited for students to seek patterns by measuring and analyzing ratios.

Agenda

Activity	Time	Description/Prompt	Materials
Introduction	10 min	• Introduce the nautilus shell to students. • Encourage students to be creative as they look for patterns in the measurements of the shells.	• Images of shells from the student handouts • Images of a live nautilus
Investigate	35+ min	Ask students to look for patterns in the shells and record measurements.	• Rulers • Digital calipers • Graph paper • Colored markers or pencils • Transparent patty paper or overhead projection sheets • Standard or digital compasses
Create	30+ min	Create a visual display and description of the patterns found in the shells. Share the findings with the class	• Chart paper, markers, graph paper, colored pencils
Discussion	15+ min	• Discuss group findings with the whole class.	

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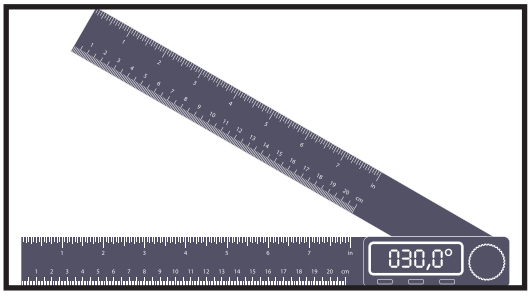
Activity

Version 1, What would the nautilus say? Does it contain a golden ratio?:
In this version we connect the Golden Ratio to the Fibonacci sequence. Consider using this video about the Fibonacci sequence created by our students from summer camp,
http://www.youcubed.org/wim/fibonacci-sequence/

We ask students to complete the Fibonacci sequence and then convert the ratios of the sequence into decimal numbers. Students should have access to calculators for this activity. The goal is for students to notice how the decimal approximations approach phi, 1.618...

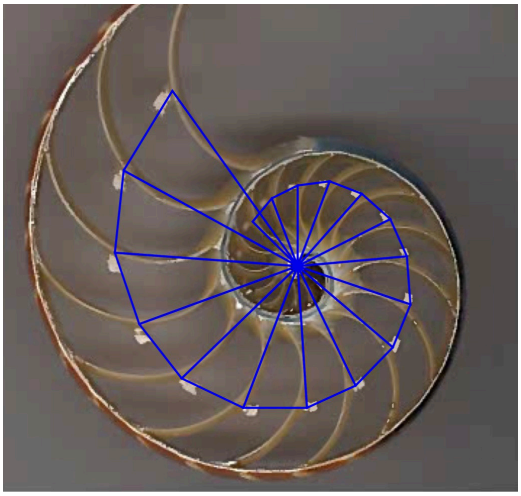
Next we ask students to look for ratios in the measures of the nautilus shell. We have provided cross sections of three different shells for students to measure and analyze. Since the pattern grows in a spiral students will need to make careful measures in a systematic way, recording and color coding to show how the measures are related. We recommend providing students with calculators and typical measuring tools found in classrooms. We love the idea of introducing students to digital calipers and protractors, tools that are used by engineers.

Digital calipers



Digital protractor

Students will choose to measure different areas of the shells in different ways. It is good to watch and listen to the ideas and methods they are choosing since it is important to identify a pattern and follow it, carefully notating and measuring the distances.



For example, one piece of student work we have seen showed students measuring from a center point they identified in the shell. The students put the shell image into a draw program on their computer and made line segments showing their pattern. Measuring the length of each segment in order provided them a table of values. They calculated the ratio of growth by dividing the length of a chamber by the length of the previous chamber.

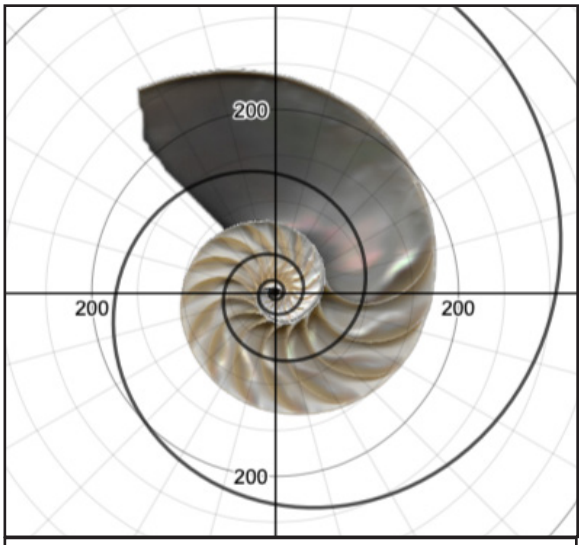
After students have had time to explore patterns in their measures we recomend you give them time to share their methods and findings with other students.

Version 2, What would the nautilus say? Logarithmic, golden, or something else?

For students ready to explore logarithmic spirals, we recomend you introduce the golden ratio phi, which is equal to $\frac{1+\sqrt{5}}{2}$

and approximately 1.618. Students may then consider whether the nautilus spiral is logarithmic or a special case where the growth factor is phi. Desmos is a great way to experiment with what spirals might best fit the shells.

Students may want to measure and look for patterns in their measurements in order to make conjectures about the growth of the nautilus shell. We recommend making different tools avaialable for use. For example, calculators, computers, digital calipers, digital protractors, compases, and rulers are good choices.



This image is a screen shot taken of some student work using https://www.desmos.com

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What does the Nautilus say?
Does it contain a Golden Ratio?

While the nautilus shell is often represented in popular culture as an example of a golden spiral, according to many mathematicians it is not. We ask, is the Nautilus golden? Can you find evidence of the Golden Ratio and spiral in the Nautilus?

The Golden Ratio is a famous and popular relationship. One common example of where it is found is the Fibonacci sequence. This sequence starts as 1, 1, 2, 3, 5, 8, 13, ... What are the next 6 terms of this sequence?

1, 1, 2, 3, 5, 8, 13, ___, ___, ___, ___, ___, ___

When you calculate the ratios of two consecutive terms in the Fibonacci sequence, the resulting decimal approximations approach the Golden ratio represented φ, is equal to $\frac{1+\sqrt{5}}{2}$. In decimal form it looks like 1.618...

Rewrite each ratio below in it's decimal equivalent and watch the magic happen.

$\frac{2}{1}=$

$\frac{3}{2}=$

$\frac{5}{3}=$

$\frac{8}{5}=$

$\frac{13}{8}=$

$\frac{21}{13}=$

$\frac{34}{21}=$

Keep going and see what happens

Your task: Is the nautilus shell an example of the Golden ratio? Can you find evidence of the Golden Ratio in the Nautilus? Use your creativity to make an argument about whether or not the nautilus is golden. Use the images of three different nautilus shells and a ruler, protractor, calculator/graphing program (desmos, geogebra,..) or any other tools you might find useful to make your argument. Make a presentation that will convince a skeptic.

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What does the Nautilus say?
Logarithmic, golden or something else?

While the nautilus shell is often represented in popular culture as an example of a golden spiral, according to many mathematicians it is not. They acknowledge that it is a classic example of a logarithmic spiral, but claim it does not have the growth factor ϕ (1.618...) required to make it the special case of the logarithmic spiral traditionally deemed the golden spiral. We ask, could an alternative frame or change in axioms give the nautilus shell a golden hue? What other patterns might be hiding in it's shell?

Is the nautilus shell an example of a golden spiral or just fools gold?

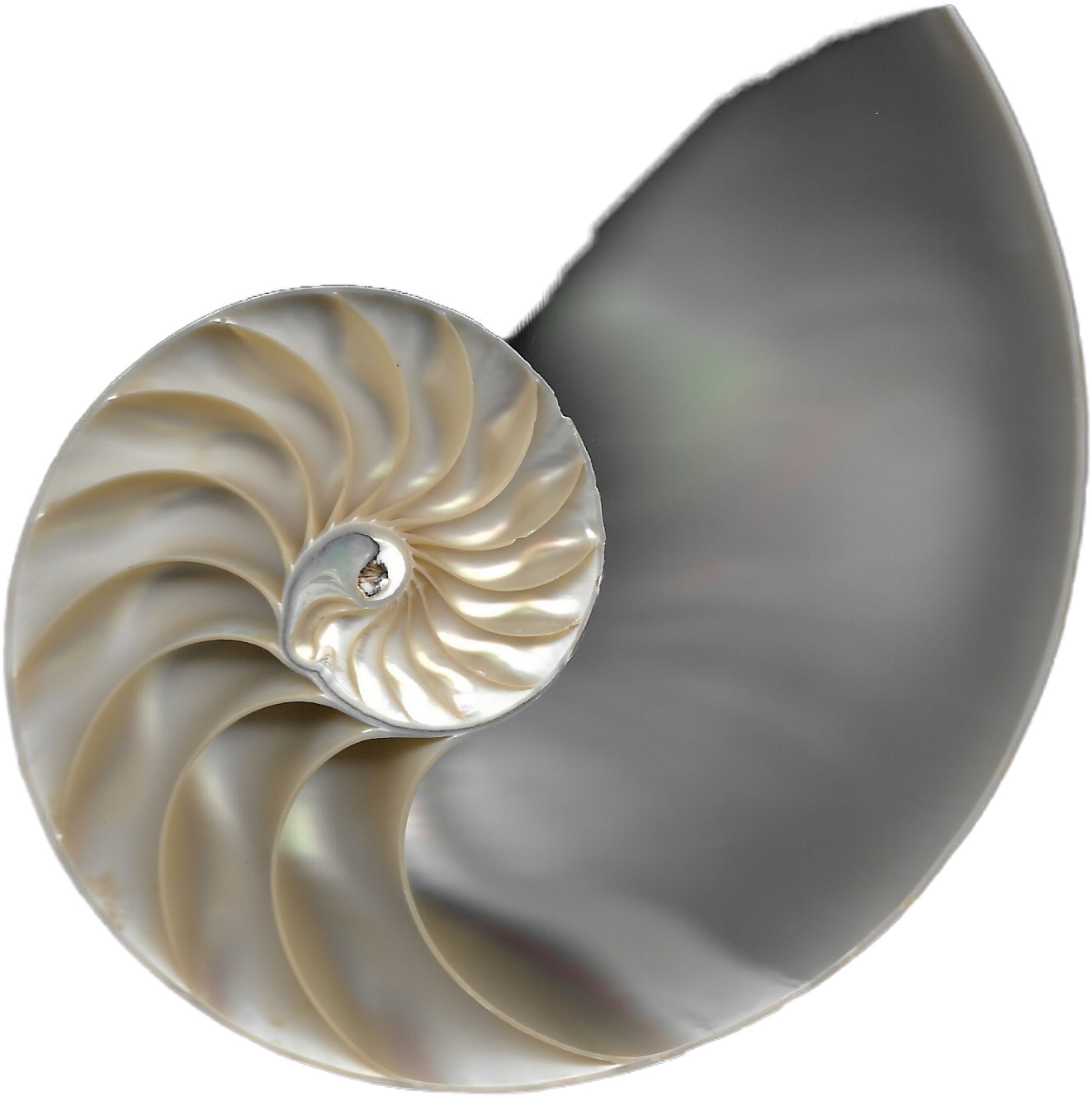
Your task: Use the images of three different nautilus shells and a ruler, protractor, calculator/graphing program (desmos, geogebra,..) or any other tools you might find useful to make your argument.

Supply List:

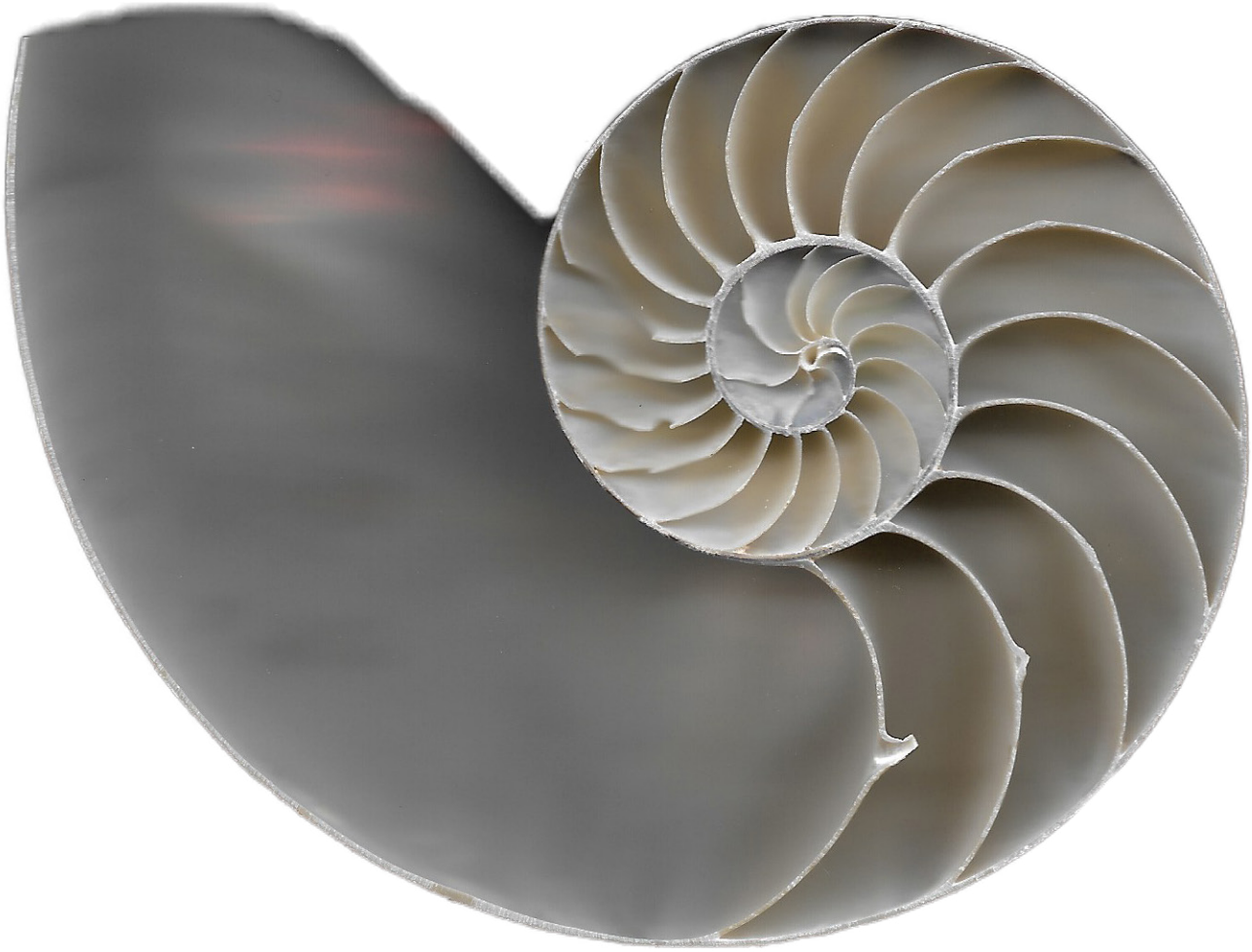
- Images of nautilus shell
- Ruler
- Protractor
- Other optional tools: compass, digital calipers, digital angle finder (protractor), tape measure, string, graph paper
- Any other tools you think might be useful

Nautilus 1

Nautilus 2



Nautilus 3



A nautilus