



Painted Cube

Grades 9-12

Introduction

In this activity students get the opportunity to see mathematical ideas, to explore patterns, to model and build, and to use algebraic thinking and generalization. It is a very rich and engaging activity, that can be used over two lessons.

Agenda

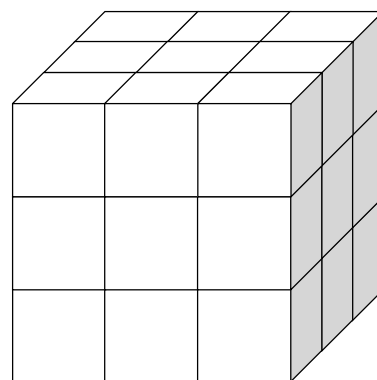
Activity	Time	Description/Prompt	Materials
Mindset Video	5 min	Play the mindset video.	Mindset video
Painted Cube	45-60 min	Explore the number of cubes with three, two, one, and no sides painted in a 3x3x3 cube	<ul style="list-style-type: none"> • 1 copy of a 3x3x3 cube for display, page 4 • Sugar cubes • Markers • Grid paper • Pencil/pen
	20-30 min	Think about the number of painted cubes in bigger cubes e.g. 4x4x4 cube and a cube of any size. How many cubes would be painted on three, two, one, and no sides?	
Day 2 of Painted Cube	20-30 min	<ol style="list-style-type: none"> 1. Give class time to bring together their results on different sized cubes in their table groups 2. Create a class chart of findings. 3. Work towards agreement on each entry. 4. Share patterns you notice. 	<ul style="list-style-type: none"> • White board • Markers
Debrief Mindset Messages	5 min	Debrief the mindset messages for this activity.	



Activity

This activity involves exploring the patterns in a cube; there are so many patterns to see that it is an ideal time for students to use tables to record their thinking.

Start by showing a $3 \times 3 \times 3$ cube that is made up of smaller $1 \times 1 \times 1$ cubes, either drawing it on the board, or projecting it. Ask the question: "If we took a $3 \times 3 \times 3$ cube and dipped it into paint, so the paint completely covered the cube, and then we took it apart and looked at the smaller $1 \times 1 \times 1$ cubes, how many small cubes would have three sides painted? Two sides painted? One side painted? No sides painted?"

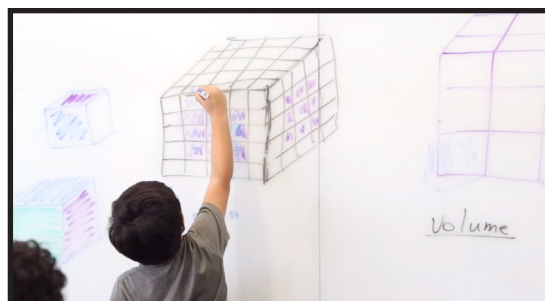


Give students enough sugar cubes to make a $3 \times 3 \times 3$ cube, and a set of colored pens, and ask students to discuss this in groups. Sugar cubes are ideal for this activity as they allow students to color them, and to look at the different painted sides.

When teams are exploring with the sugar cubes give them space to come at the problem from any direction that makes sense. Give them time to work out how to color-code the sugar cubes. Encourage students to record their findings in a table. Decide if you will give any direction about the orientation and labels to use in the rows and columns of the table.

When groups have finished investigating a $3 \times 3 \times 3$ cube ask them to think about a $4 \times 4 \times 4$ cube, and other sized cubes. Ask students if they can make predictions about bigger sized cubes, such as $5 \times 5 \times 5$.

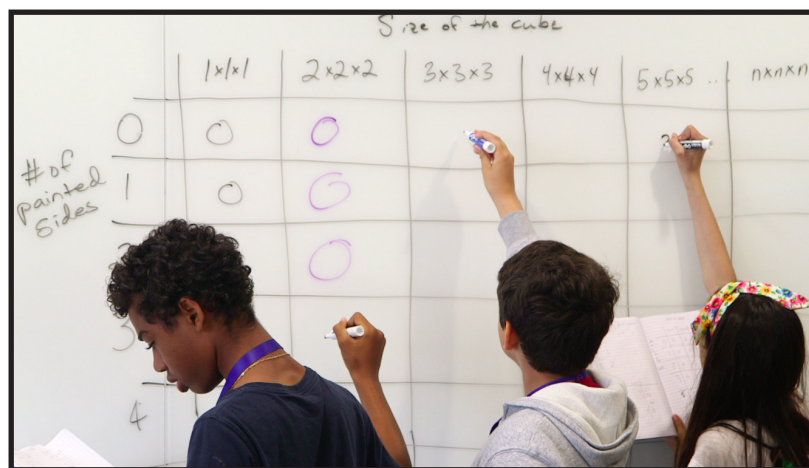
Towards the end of class time pull the students together for a whole class discussion. What did the students find out? How did they know the number of sides painted? Encourage students to justify their thinking.





Possible Day 2

In our youcubed summer school we moved from the initial explorations of different sized cubes in groups to a whole class discussion that was very engaging and challenging for students. In the whole class discussion we asked students to come to the board and record any results they had, and then we worked to complete the table as a whole class.



To prepare for the whole class discussion put the outline of the table on the board. Ask any group to come to the table and record a result they found. As students record their findings encourage them to add anything they found in the table and to feel free to add columns or rows and disagree if they notice something different than another student.

Encourage students to discuss each entry until there is agreement. Give them space to explain their thinking and ask each other for justifications. If students are having a hard time with their justification, ask them how they see it, suggest they draw their thinking and give reasons based on their sugar cube model, a visual diagram, or a pattern.

In our summer school class students struggled to make sense of some of the answers, and some wrong answers were put into the table, that we left for other students to notice. The group discussion was very productive – students saw the mistakes, which gave us opportunity to celebrate the mistakes, and they struggled to find all the patterns, which we also celebrated.

Remind students of the importance of visualizing and drawing in mathematics, and the power of fingers for representing numbers in the brain. Give specific examples of individuals and teams about their contributions to their team during the exploration time with their teams and reaching agreement on the information in the table with the class. Include moments when students made mistakes, gave a good explanation based on a visual, challenged an idea shared, etc.

Extensions

Write expressions for the number of cubes with three, two, one, and no sides painted for a cube of any size.

