



## Game of Hex Community College

### Introduction

Hex is a two-person game with very few rules but a lot to think about. Students have an opportunity to explore different approaches to see if they can find a winning strategy. Mathematicians have been doing so in the over half a century since its invention and this problem gives students a chance to approach it in the same way mathematicians do. Finding a winning strategy is a key part of game-theory a branch of mathematics often used in areas such as Economics.

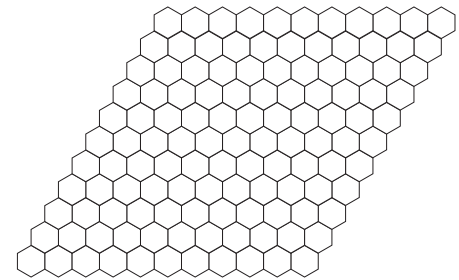
### Agenda

Activity	Time	Description/Prompt	Materials
Mindset Message	10 min	Play the mindset video.	Mindset Video
Play	10 min	<ul style="list-style-type: none"> <li>Introduce the Game of Hex to the students.</li> <li>Have students work in pairs and play the game with each other. Is it possible for the game to end in a tie? Why or why not?</li> </ul>	<ul style="list-style-type: none"> <li>Game of Hex handout</li> <li>Counters of two different colors (or color pencils)</li> </ul>
Investigate	30 min	<ul style="list-style-type: none"> <li>Investigate different size boards. Does the size of the board change the conclusions you had drawn before?</li> <li>Can you find a winning strategy for either the first player or the second player?</li> </ul>	<ul style="list-style-type: none"> <li>Game of Hex handout</li> <li>Hexagonal grid paper</li> <li>Counters of two different colors (or color pencils)</li> <li>Maths Journal</li> <li>Pencils</li> </ul>
Discuss	15 min	Share conjectures and justifications with the rest of the class. Work on a joint proof of one of the questions the class has been working on. Visual proofs are encouraged.	
Debrief Mindset Message	5 min	Debrief the mindset messages for this activity.	



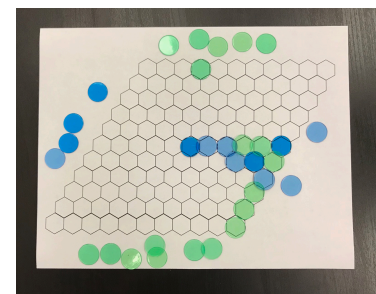
### Activity

Introduce Hex to the class. Display the game board and tell them the object of the game. Share some history of the game with students. Let students know Piet Hein, a Danish mathematician, inventor, and poet, invented the game. He was a student at the time he developed the game in 1942. That same year, Politiken, the leading Danish newspaper, published a version of the game and it became hugely popular in Denmark. Hex was then brought to the U.S. by way of Princeton where they called it “John” in part because it was often played in the hexagonal tiles in the bathrooms there<sup>1</sup>. Since then it has become a beloved game of mathematicians everywhere, with many papers and theorems written about it. This activity gives students a chance to play the game and make conjectures about strategies and outcomes like a mathematician.



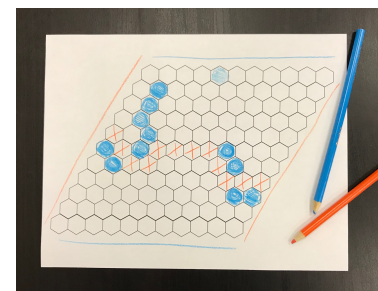
Hex gameboard

Have students select partners and try the game out with the initial question of whether or not it is possible to end the game of Hex in a tie and why. Once they’ve had some time to familiarize themselves with the game and think about this question, have two sets of partners form a group of four to discuss their conjectures around the question about the game ending in a tie.



Hex game being played with counters

Next invite them to explore different sized game boards. Give students copies of hexagonal grid paper, so they can draw their own grids of multiple different sizes. Encourage them to investigate the question of whether the size of the board has any effect on the game. Can they find a winning strategy for the game in the standard size board and other size boards? Mathematicians have found winning strategies for some size boards and not others, so this is a good chance for students to work, like mathematicians themselves, on unsolved problems.



Hex game being played with colored pencils

Bring students back together once they’ve had plenty of time to explore the game and the questions posed. Remind them it’s okay if they’re not completely done with finding an answer because mathematicians often work on problems over days, weeks, months, and year. Invite students to share their thinking, full ideas or partial ones, and to construct a proof as a class for a question they choose. Encourage students to put their different ideas together and build on those until they can answer the question they are working on in a way that satisfies them.

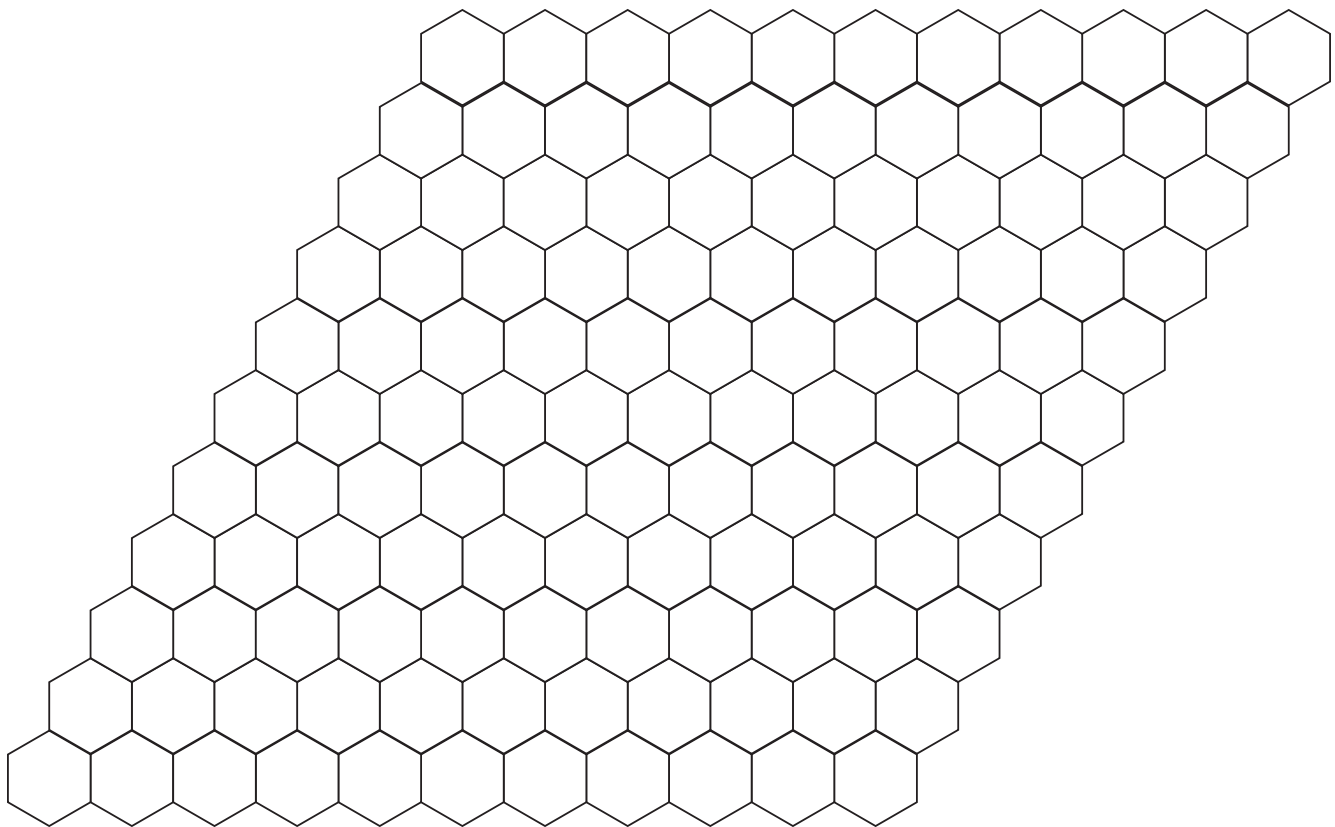
<sup>1</sup> Martin Gardner. Mathematical Games: Concerning the game of Hex, which may be played on the tiles of the bathroom floor. Scientific American, page 145ff, July, 1957.



## Game of Hex

The goal of the game of Hex is for each player to make an unbroken chain with their own pieces connecting the two opposite sides of the board marked with their color. The standard game of hex is played on an 11x11 hexagonal tiled rhombus shaped game board, as you can see below. If you want to play on your own against a computer or play with another person digitally visit <http://www.lutanho.net/play/hex.html>

Hex Gameboard:



Choose a partner and play the game of Hex using the following rules:

- Each player selects a color and a side of the board.
- Take turns putting a counter or marking any unoccupied hexagon on the game board.
- Continue taking turns until one player creates an unbroken chain connecting the two opposite sides of the game board marked with their color.

Consider the following questions:

- Is it possible for the game to end in a tie? Why or why not?
- Does the size of the board change the conclusions you had drawn before?
- Can you find a winning strategy for either of the players?



## Hex Gameboard

