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Towers
Day 4

Introduction

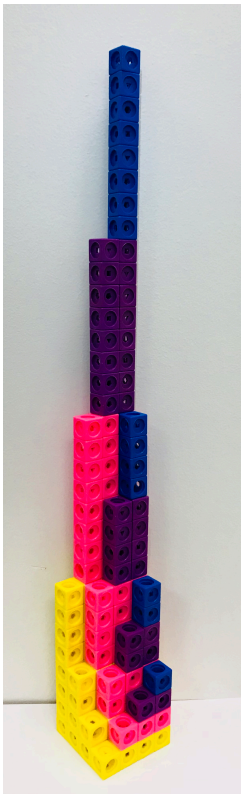
This activity allows students to explore a 3-dimensional growth pattern. Typically the growth patterns students are familiar with are 2-dimensional growth patterns. This growth pattern is also distinctive in that there are different ways for students to see the growth. Students may come up with different solutions depending on how they see the pattern. This gives students an opportunity to share their assumptions for how the pattern is growing before describing their solutions.

Agenda

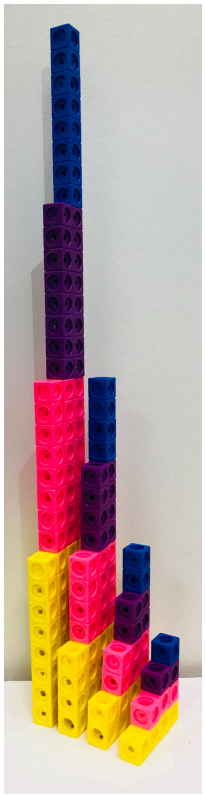
Activity	Time	Description/Prompt	Materials
Mindset Message	10 min	Play the mindset video, ***Strategies for Learning Maths***, https://youcubed.org/weeks/week-4-grade-9-12/	Mindset Video day 4, ***Strategies for Learning Maths***
Explore	20 min	• Introduce the problem. • Give students time to explore Towers.	• Towers Pattern: Case 5 • Towers handout • Maths journals • Pencils • Colored pencils or pens • Colored cubes: yellow (60), pink (45), purple (30), blue (15)
Discuss	10 min	• Invite students to share their findings. • Have students discuss the type of growth they think the pattern is; linear, quadratic, exponential or something else.	
Debrief Mindset Message	5 min	Ask students to reflect on all the Strategies for Learning Mathematics from the video: 1) Draw it out, 2) Teamwork, 3) Experiment, 4) Look for different resources, 5) Start with a smaller case. Highlight some moments when you saw individuals and groups using these strategies or ask students to share when they used the strategy or saw someone else use the strategy.	

Activity

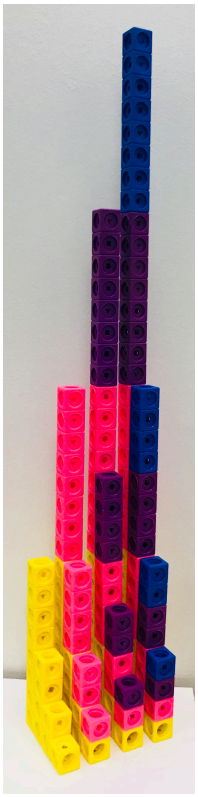
Introduce the problem to students and project the picture of case 5. Give them colored cubes for building their own tower along with colored pens or pencils for sketching. Allow them to work in partners or groups as they explore how the pattern is growing. Let them know that there are different and defensible ways to see case 5 and the growth in the pattern, so it will be important to share with each other the decisions behind their conjectures. Remind them of the strategies they just saw discussed in the video and to use some of them while they explore the tower pattern.



Case 5
View 1



Case 5
View 2



Case 5
View 3

As students are working, encourage them to share with each other how they see the pattern growing and give their reasoning. Invite them to give clear descriptions to each other. Encourage them to ask questions of each other to understand how their peers are seeing the pattern growing. Listen for student's explanations as they describe how tall the tallest tower will be in case 5, in case 100 and in the n^{th} case. Encourage students to use representations that make sense to them. The representations students will likely use are describing in words, sketching well-labeled diagrams, and writing numeric and algebraic expressions. Remind students they can choose the representations they use and do not need all three.

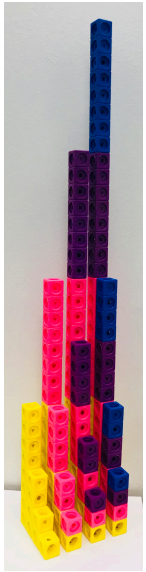
When partners or groups see the pattern growing in different ways, they may get a different answer for

what case 100 and any number n would look like and the number of cubes it would have. When you hear this happening in a group encourage them to be open to each other's ideas and be skeptical. When they ask if there is more than one answer for case 100 and n, ask them what they think; if they think there could be more than one way to see the pattern growing and if that means there could be more than one answer. Support partners and groups in a discussion about what agreements they want to make or if it makes sense to have multiple answers.

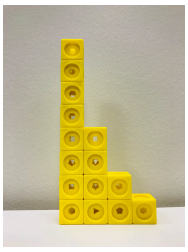
Once students have an opportunity to explore all of the questions, bring the class together to share their findings. Invite students to share different ways they saw the growth. Encourage them to be skeptics. Share with them that part of being a mathematician is to ask questions and to give clear explanations with reasons.

Once students have shared all the different ways they see the shape growing, ask students to share how many cubes are in the tallest tower and how they know. Move the conversations to students sharing what the n^{th} case would look like and how it would be different depending on how you saw the pattern growing. Have students share and explain how many cubes in the n^{th} case and what it would look like, encourage them to share all of the different representations they used: visual, table, graph, algebraic expression. Discuss the representations shared with the class. Have a conversation about the connections between the different representations of the n^{th} case and the number of cubes in the tallest tower in the n^{th} case. Ask students how their representation relates to the expression for the number of cubes in n^{th} case.

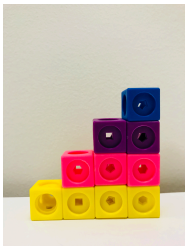
Ask students to share their ideas about how each slice of the tower is related in the 5^{th} case. Encourage students to compare the slices moving from 'left to right' and 'front to back'. Invite students to make conjectures about how to generalize the number of cubes in the last row and column ('left to right' slice and 'front to back' slice) of the tower in the n^{th} case.



Case 5
View 3



'Left to right' slice from case 5



'Front to back' slice from case 5

If students see the pattern growing differently there will be different answers for the 100^{th} and n^{th} cases. This creates an opportunity to share the idea with students that there can be different answers to the same problem depending on how you see it. This is common in mathematics and is why it is important to be clear about how you see something.

Invite students to share the type of growth of the pattern. Have them make conjectures about the pattern growing linearly, quadratically, exponentially, or something else. Encourage them to connect the way they see the pattern growing and the type of growth. Make space for students to consider the different ways of seeing the pattern and their conjectures about the type of growth the pattern represents. The type of growth will depend on how they imagine the pattern growing. Mention to students that as long as they can reasonably justify their conjecture it is possible there are different answers.

Extension

- Make your own 3-Dimensional growth pattern and describe how it is growing.

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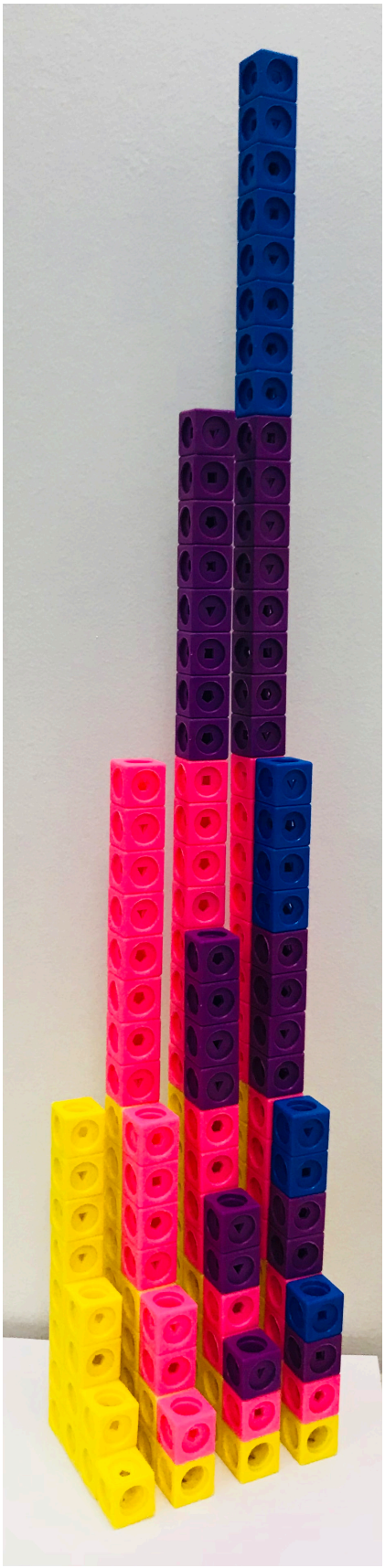
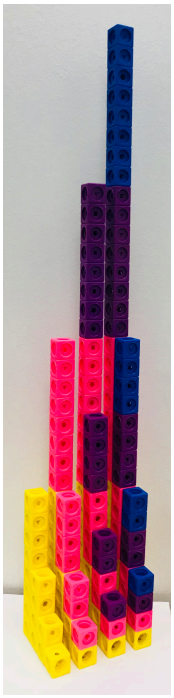
Towers Pattern

Case 5



Towers

Here is case 5 of a tower pattern built using multilink cubes:



Case 5

How do you see this pattern growing? What do you think case 1, 2, and 3 would look like?

Consider the following questions about the tower:

- What would the tower in case 100 look like?
- What do you think the n^{th} case looks like?
- How many cubes do you think are in the tallest tower in the n^{th} case?
- How do you see the different slices of the tower related to each other?
- What kind of growth is represented by the tower pattern? Linear, quadratic, exponential or something else?