

Apple Orchard

Week 3 - Days 1, 2, & 3

Introduction

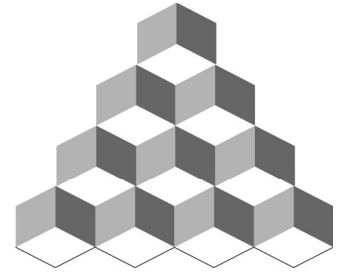
This activity invites students to explore a pattern that includes both linear and quadratic growth in the same relationship. We first saw this pattern in the PISA released items in 2006 (<https://www.oecd.org/pisa/38709418.pdf>), and we were inspired to create a visual activity. This pattern is a wonderful example of linear and quadratic growth. Students will be asked to identify and extend the patterns as they create graphs, tables and written descriptions in order to generalize the functions. This activity begins with a visual number talk where students quantify the number of x's and dots in the third case.

Connection to CCSS

MP 7
MP 8
HSA.CED.A.2
HSF.LE.A.3

Agenda

Activity	Time	Description/Prompt	Materials
Launch	10 min	- Share the picture of the pattern with students as you would in a dot card number talk. - Ask them how many x's and dots there are in the picture without counting one at a time. - Have students share their different ways of seeing and record their ways of seeing.	Apple Orchard Visual for case #2 to project
Explore	30+ min	- Give students the Apple Orchard Handout and let them explore the pattern in groups. - Ask students to record their multiple representations: graph, tables, equations, visual patterns and written descriptions on posters	- Apple Orchard Handout - Graphing Technology - Maths Journal - Graph Paper - Poster paper - Colored pencils or pens
Discuss	20 min	- Invite students to share their conjectures. Encourage them to share what they notice within representations and across representations. - Introduce linear and quadratic terms and encourage students to use them in the conversation.	- Whiteboard/poster paper - Markers

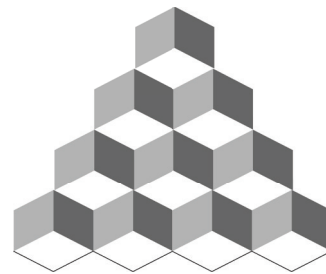


Agenda continued

Activity	Time	Description/Prompt	Materials
Explore	10+ min	Ask students to explore what would happen in the negative two (-2) case.	Maths journal
Discuss	10+ min	- Have students put their findings on the board or poster paper (or extend what is currently there). - What do you notice with each of the growth patterns in the different representations when the case number is negative? Why is this happening?	
Extend		- Make your own visual pattern or patterns that contain both linear and quadratic growth. What is your justification for how you know one pattern is linear and the other quadratic? - Ask students to prepare a poster showing their pattern so other students can extend the patterns, create tables and graphs and generalize the functions.	- Maths journal - Poster paper - Colored pens or pencils
Reflect	5 min	What do you notice about the differences in linear and quadratic growth across multiple representations?	Maths journal

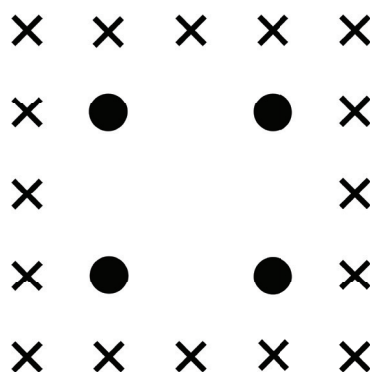
To the Teacher

We use this activity to continue building understanding of variables and the different ways we can represent patterns with tables, graphs, written descriptions and generalized expressions. By launching the pattern in the format of a dot card number talk students can share the different ways they see the growth. This allows students to hear the different ways others see the growth. This continues to communicate the importance of understanding different approaches and perspectives. If you would like to read more about dot card number talks see the dot card number talk lesson plan in week one, visit this link: [<https://www.youcubed.org/downloadable/algebra-week-1-productive-class-culture-lesson-2/>]



Launch

Project the pattern and ask students how many x's and dots they see in the pattern without counting them one at a time. Only show the diagram for a short period of time so students do not have time to count. Ask students to share how many x's and dots they came up with. Make sure to ask if someone got a different answer to reinforce the possibility of multiple answers. Once students have shared all the answers they have (and recording these on the board), ask students to share how they saw it. You can show the case number again so students can reflect on their answers. Record their different ways of seeing like you would a dot talk. You can see Jo introducing a visual pattern like she would a dot number talk in this video of the Border Problem. [<https://www.youcubed.org/resources/border-problem-video/>]



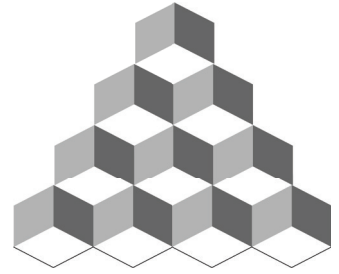
Explore

Give students the Apple Orchard Handout and invite them to explore the problem in their groups. Encourage them to share with each other their different ways of seeing the relationship's growth as the apple orchard changes in size. Next, have students work to answer the questions on the handout. Give students space to answer the questions using any strategy they like. Invite students to show off the pattern with multiple representations: the visual pattern, tables, graphs, written descriptions and expressions.

As students continue the pattern they will see that the quadratic growth (the dots) is catching up to the linear growth (the x's) and then surpassing it. Some will discover this as they graph it in graphing technology or on paper, others will see it in the table, and others will notice it in the visual pattern. You may ask students, "What happens to the number of x's and the number of dots as the case numbers increase?"

Discuss

Have blank tables, blank coordinate graphs, space for equations and an area for students to continue the pattern on display so students can share what they have found. Ask students to come up to the board and fill out information for one of their representations. Students should fill in their information without judgement and should be encouraged to share their answers even if they are different. This allows for great conversation as students come to a consensus as a community of maths learners. This is the time to celebrate mistakes and learning. If students come up with different equations (from seeing the pattern in



different ways) encourage students to discuss the different representations and work through determining which are appropriate representations of the pattern.

Start the conversation with, “What are the conjectures you made in your groups?” “What did you notice?” Have students come up to the board and share different things they noticed about the differences in the growth patterns across the multiple representations. Invite students to highlight connections between representations. If students aren’t getting specific in their conjectures within the graph and table, then begin to ask more specific questions. “What did you notice about the growth differences between these two tables?” If students haven’t used the terms quadratic and linear in their conversation, introduce these terms into the conversation. If there are comparisons between the two different types of growth that students haven’t shared, you can highlight these by saying, “Students in another class noticed... what do you think?”

Explore

At the end of the discussion ask the students what would happen with these multiple representations if we continued the case numbers in the negative integers. Make space for students to choose the representations they want to use to explore this question. Share with students the idea that when modeling relationships with mathematics it is possible to find functions that are related but not always exact and that by adding constraints the model can be made more accurate to fit the relationship.

Discuss

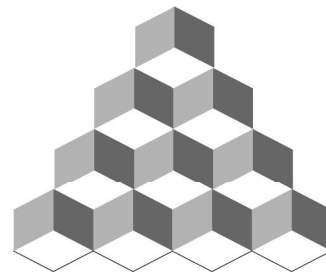
Have students extend their tables and graphs on the poster paper or the whiteboard. What do they notice about what happens with each of the growth patterns in the different representations when the input is negative? Why is this happening?

Extend

- Make your own visual pattern or patterns that contain both linear and quadratic growth. What is your justification for how you know one pattern is linear and the other quadratic?
- Ask students to prepare a poster showing their pattern so other students can extend the patterns, create tables and graphs and generalize the functions.

Look-Fors

- How are students seeing the pattern growth in different ways? Notice the different ways students are connecting the size of the visual to the number of conifer (x’s) and apple trees (dots). Are they sharing these different ways of seeing with each other? Notice if they apply their different ways of seeing from the launch to deciding how the pattern grows. They may choose to look at the pattern in a different way when they begin to enlarge it and reduce it.

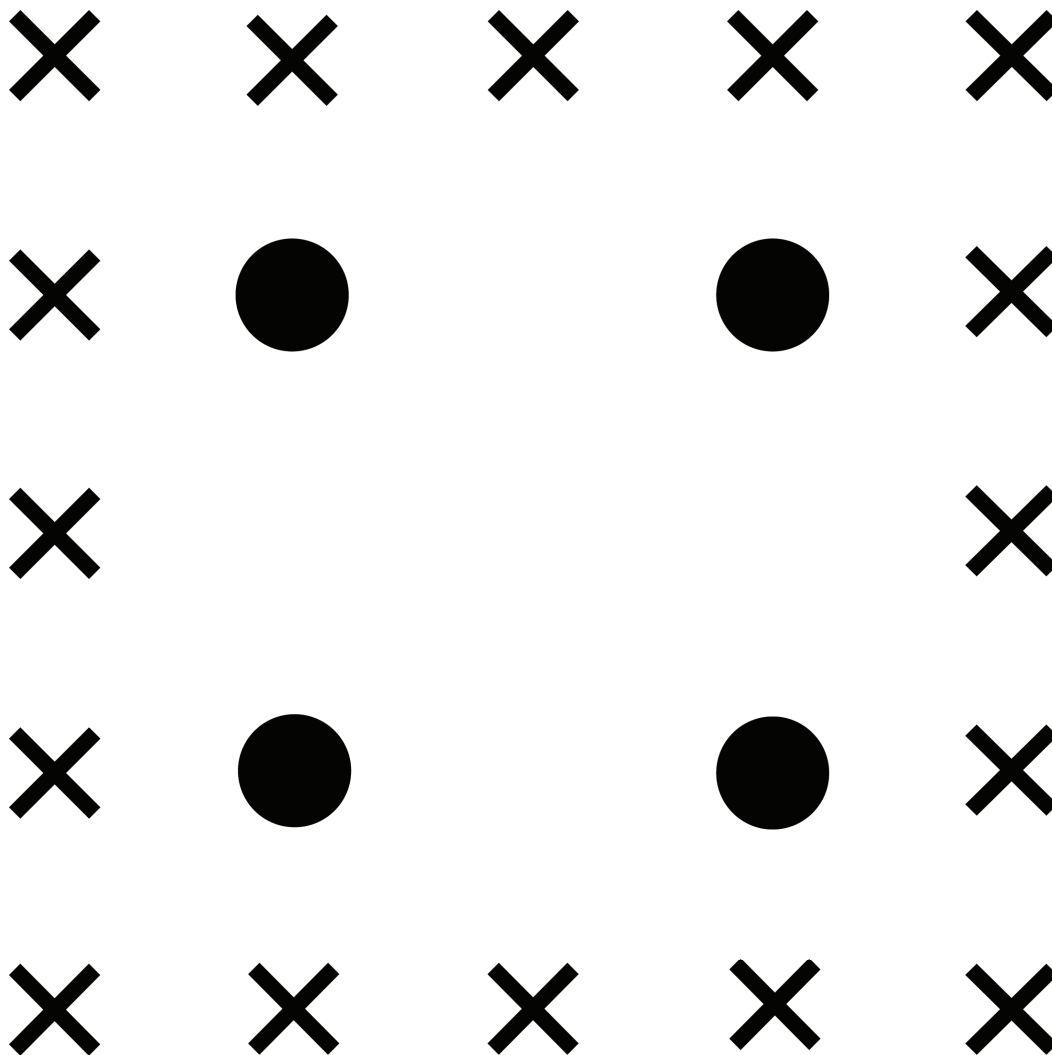
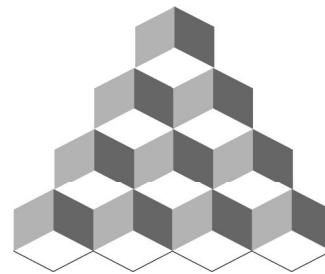


- How are students justifying their conjectures? Notice which representations students are more drawn to as they begin to make conjectures about how the patterns are growing. If students only look at small cases they may not get the full picture of what is happening with the growth of the two different types of trees. Encourage students to extend their visuals, tables, and graphs to larger cases so they can see how the two are growing differently.
- How are students noticing the differences between the two types of growth? Listen to what students are saying as they compare the two different types of growth. What do they notice between the two types of growth of the pattern in the tables, the graphs, the visuals and the equations? How do they explain and highlight the differences? Encourage students to use color, arrows, and words to highlight these differences.
- How are students making connections with previous work? As students are discussing the growth, notice if they make connections with what they have learned from previous growth patterns. Do they make connections with the border problem? Do they notice that the two patterns are growing in different ways? Are they making statements like, "This reminds me of (a specific activity or pattern)."
- How are students creating graphical representations? When students create graphical representations notice what strategies they use. Some students will make detailed graphs with labeled axes and exact coordinates for the figures they can see or entries in a table. Other students will use graphing technology to get the shape of the graph and then make a sketch with some important labeled points. All approaches will be helpful in developing an understanding of quadratic growth and graphs. Notice if some students include negative inputs on their graphs. Also, encourage students to extend their graphs. They may need to draw the graph a couple different times to be able to see what is happening over time with the growth.

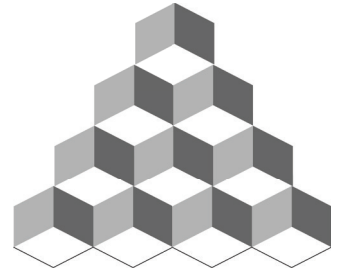
Reflect

What do you notice about the differences in linear and quadratic growth across multiple representations?

Apple Orchard Visual

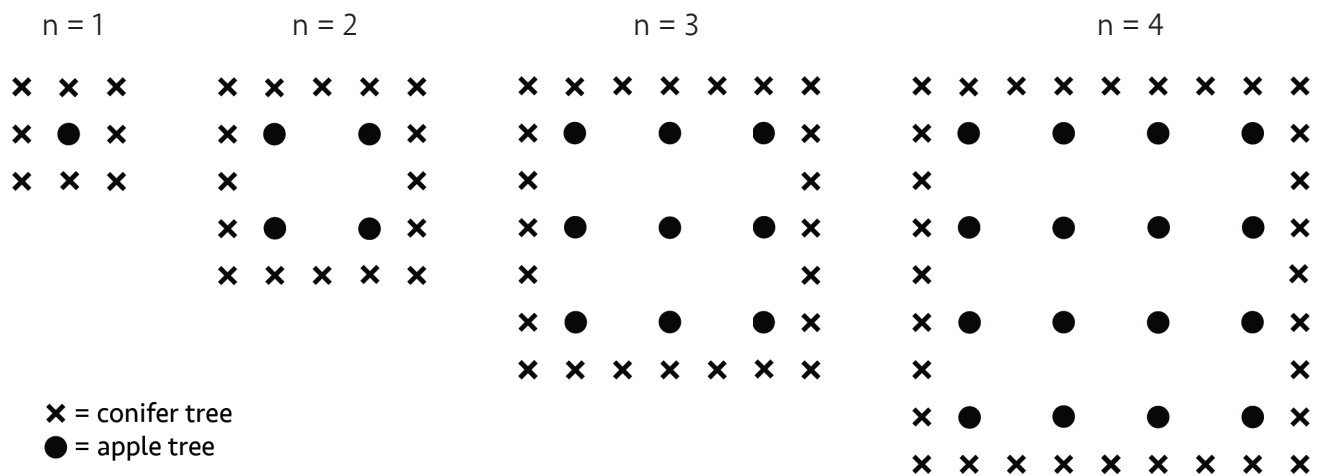


The Apple Orchard



A farmer plants apple trees in a square pattern. In order to protect the apple trees against the wind he plants conifer trees all around the orchard.

Here is a diagram of the situation where you can see the pattern of apple trees and conifer trees for the first 4 cases:



How do you see the pattern growing? Extend the pattern by drawing more cases.

Suppose the farmer wants to make a much larger orchard with many rows of apple trees? As the farmer makes the orchard bigger, following this pattern, which will increase more quickly: the number of apple trees or the number of conifer trees? How do the growths compare to each other? Justify your answer using multiple representations: the visual pattern, equations, tables, written descriptions and graphs.

Write a description of how you would determine the number of conifer and apple trees in any case number.

Write general expressions so you can calculate the number of conifer and apple trees in any case number.