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## Introduction

In day 2 we have a video and an activity. The activity invites students to investigate a really interesting representation of numbers, created by Stephen Von Worley, that fascinates children and adults alike and gives students an important opportunity to understand numbers and to think visually about them. The timing for the lesson is given after my description of the tasks.

## Video

The video shares some important new research on the power of engaging with numbers and symbols visually, which involves brain crossing. Some people have been given unhelpful ideas that they are visual learners or not visual learners. The video explains that it is helpful for all students to think visually about mathematics, and today's activity is a perfect opportunity for this.

Activity:
As the introductory video explains we now know that when students think of maths visually as well as with numbers and symbols they are crossing the brain, using different pathways, and that has been found to increase the power of maths learning. This activity is a perfect way to encourage brain crossing and deep understanding. When we first saw this representation of numbers we were intrigued and when we looked further we saw that the representations of the numbers highlight their composition really nicely. Both teachers and students who have seen this visual have loved it and wanted to spend time with it. It is engaging for students of all ages and achievement levels. In our trials we found it most useful just to ask students first what they see. The classrooms soon started buzzing with students noticing that "all the circles are prime", and that number pictures show factors. This is a great activity for colour coding, as students can use colour to show the factors. Some students will see that the primes are all in diagonals on the table but one is interrupted by the number 25 , are they curious about this?

Have students sit in groups to conduct their investigations, so that they can talk and compare notes. We think it is fine for students to work on a pattern on their own or with others.

Other interesting questions to ask, that will get students' brains working hard, are what would other numbers look like if we followed the method of drawing? E.g. what would 36 and 37 look like? When students have explored patterns for a while, ask them to present their ideas to each other. This will probably take a whole lesson but if you have more time a complementary number investigation that we like is 'consecutive numbers'.

## Consecutive Numbers:

There are many different versions of consecutive number investigations and we are sharing a few of them. Our favorite version to go with the number visuals activity is one that uses a hundred chart. If you are a middle or high school teacher don't assume that the hundred chart is for younger students since this activity extends to algebraic representations and the hundreds chart helps stu-

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | $n$ | $n+1$ | $n+2$ | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | $m$ | 34 | $m+2$ | 36 | $m+4$ | 38 | 39 | 40 | dents see and understand expressions. Often students are shown consecutive number problems and many don't connect the expression $n, n+1$ and $n+2$ to actual numbers. This is especially challenging when students are shown the expressions $n, n+2$ and $n+4$ and told that this represents three consecutive odd numbers. Using a hundred chart really helps students understand the abstract representations.



| Activity | Time | Description/Prompt | Materials |
| :--- | :--- | :--- | :--- |
| Day 2 Video: <br> Brain Crossing | 2 min | Video <br> https://www.youcubed.org/wim-day-2/ | Number Visuals <br> Num min |
| 1. Write the number above each representation. <br> 2. What do you see? <br> 3. Use colours to show patterns. | Paper, pencil/pen <br> - <br> - <br> Coloured pencils/markers <br> Number Visual handout, <br> page 3 <br> Number Visual Activity <br> handout, page 4 |  |  |
| Group <br> Presentations | 20 min | Ask students to share any patterns or other inter- <br> esting observations |  |
| Closing | 5 min | Review the key concepts: Math learning is best <br> when we have opportunities to make connections <br> between pictures and numbers. It is good to draw <br> and to try to understand mathematics visually. |  |

Extensions:
Number Visuals

- Draw the numbers 36 and 37.
- Create your own visualization for the numbers 1-20

Consecutive Numbers, page 5

- Hundred Chart, page 6
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## Day 2: Number Visuals



1. Write the number that each visual represents on your number visuals handout.
2. What do you see in the number visuals? Do you notice anything interesting about the way numbers are shown? Share your findings with your group members and discuss them together.
3. Look for interesting patterns. You may find it useful to use colours to highlight them. Describe some of your findings and share with your group members.

## Day 2: Number Visuals

## Consecutive Numbers

The number 12 can be written as a sum of consecutive numbers,

$$
3+4+5=12 .
$$

Another example of a consecutive number sum is 3 since

$$
1+2=3 .
$$

Can all numbers be written as sums of consecutive numbers? Can some consecutive number sums be written in more than one way?

Using the hundred chart circle three numbers in a row (horizontally) and add them. Try this with several sets of numbers. Do you see a pattern? Does your pattern work for every group of three consecutive numbers? Write a convincing argument.

Using the hundred chart circle four adjacent numbers to form a square. If you add the diagonals what do you think will happen? What does happen? Does this work for every group of numbers in this pattern? What do you wonder? Write a convincing argument.

Using the hundred chart circle four adjacent numbers to form a square. If you multiply the diagonals what do you think will happen? What does happen? Does this work for every group of numbers in this pattern? What do you wonder? Write a convincing argument.

Day 2: Number Visuals
Hundred Chart

| 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 11 | 12 | 13 | 14 | 15 | 16 | 17 | 18 | 19 | 20 |
| 21 | 22 | 23 | 24 | 25 | 26 | 27 | 28 | 29 | 30 |
| 31 | 32 | 33 | 34 | 35 | 36 | 37 | 38 | 39 | 40 |
| 41 | 42 | 43 | 44 | 45 | 46 | 47 | 48 | 49 | 50 |
| 51 | 52 | 53 | 54 | 55 | 56 | 57 | 58 | 59 | 60 |
| 61 | 62 | 63 | 64 | 65 | 66 | 67 | 68 | 69 | 70 |
| 71 | 72 | 73 | 74 | 75 | 76 | 77 | 78 | 79 | 80 |
| 81 | 82 | 83 | 84 | 85 | 86 | 87 | 88 | 89 | 90 |
| 91 | 92 | 93 | 94 | 95 | 96 | 97 | 98 | 99 | 100 |

