Journal of Mathematical Behavior 41 (2016) 172–178

Contents lists available at ScienceDirect

The Journal of Mathematical Behavior

journal homepage: www.elsevier.com/locate/jmathb

Editorial

Designing mathematics classes to promote equity and engagement

1. Introduction

I still remember the moment when I sat down with my graduate students and we decided to take on the mathematics teaching of a 5-week summer school program. We all agreed that we had learned a lot from research since we had been classroom teachers, about the importance and nature of the different dimensions of good teaching. We welcomed the opportunity to put what we had learned into practice. The challenge was there and we agreed to take it on.

In this special issue we present a mathematics teaching intervention that we designed and taught to four classes of disaffected 7th and 8th grade students over one summer. The class we designed was an 'exploratory algebra' class that focused on mathematical 'practices' (RAND, 2003; CCSS, 2010). The results of the experience seem important to share, especially at a time when so many students are failing math, particularly in low income, urban communities such as the one in which we taught the intervention (Boaler, 2013b; Boaler & Foster, 2014). One of the most important statistics we have read in recent years tells us that approximately 60% of college students in the US are in 2-year colleges and 75% of them are required to take remedial math – basically repeating the courses they took in high school. Shockingly only one in ten of the students pass the remedial courses, the rest have to leave without any college degree (Silva & White, 2013). This math failure does not start in college – it starts much earlier and we knew that the students we were going to teach that summer had already started out on the long path of failure. We wanted to change their direction.

During the 5-week intervention the students experienced mathematics in ways that they had never before experienced it. This was because we taught 'multi-dimensional' mathematics (Boaler, 2009) engaging students in the disciplinary practices such as problem solving, reasoning and constructing arguments, that are now enshrined in the Common Core Practice Standards. This broader form of mathematics, closer to the discipline of mathematics (Boaler, 2013b), served to re-engage the students, as one of the students, Sabrina, reflected:

> It's like the way – the way our schools did it is like very black and white, and the way people do it here, it's like very colorful, very bright.

The mathematics with which the students engaged was powerful, interesting and as Sabrina said, colorful enough that it became a new and exciting subject for many of the students. In this special issue we describe the design of the teaching that enabled this broader, colorful mathematics to emerge, as well as the research results that reveal the impact upon student learning shown through a standardized algebra test.

2. Organization of the special issue

The remainder of this paper describes the teaching intervention we designed, including detail on the teaching decisions that is of the grain size that may be productive for other teachers. In the paper that follows (Paper 2, this issue) we analyze the learning of all of the students, by looking at assessment data, student interview and survey data, to consider the impact of the intervention upon student learning and achievement. In the third paper (Paper 3, this issue) we zoom in on a particular case of student learning through engaging in the mathematical practice of representation. The fourth paper (Paper 4, this

http://dx.doi.org/10.1016/j.jmathb.2015.01.002

Table 1
Student ethnicities.

Hispanic	39%
White	34%
African-American	11%
Asian	10%
Filipino	5%
Native American	1%

Table 2
Student prior grades.

A or B	40%
C	20%
D or F	40%

issue) considers the critical role played by student agency in students' engagement, and the fifth paper (Paper 5, this issue) illustrates the importance of mathematical aesthetics and problem posing.

3. The teaching intervention

3.1. The students and teachers

The instructional team consisted of a mathematics education professor (myself) and four graduate students, all with prior teaching experience. The classes were 90 min long and they met for four days of the week (Monday–Thursday).

The students who attended summer school were an ethnically, culturally, socially and academically diverse group and they had just finished either 6th or 7th grade. Their ethnicities are represented in Table 1.

We chose to group students heterogeneously and the students' prior mathematics achievement was distributed across the achievement spectrum. Table 2 shows the students' grades from the last math class they took, in the spring semester.

Despite the breadth of achievement the vast majority of the students viewed mathematics negatively. On the first day of class we administered a survey that asked the students whose idea it was to come to summer school and asked them: "When you think of math what comes to mind?" Ninety per cent of the students had been 'made' to come to the summer school by teachers and parents and 'boring' was the most commonly used adjective to describe math ($n = 94$). The main reasons given by the students for not wanting to be in the class were: (1) it would be boring; (2) they would rather socialize with friends; and (3) it would be an unnecessary experience. A review of the student referral forms completed by their previous math teachers showed that most of the students had been referred to the summer school class because they were regarded to be mathematically weak.

On the first day of class the students' lack of enthusiasm for the 5-week math class became clear. Many of the students sloped down in their chairs, with their hoods up, looking disinterested, while others sat and chatted with friends ignoring requests from the teachers to attend to the mathematics being taught.

The content we focused our teaching upon was algebra, as we knew that this would be important for students, and most had not met algebra before, or they had met it procedurally. Rather than present algebra as an end point – something they "solved for" – we chose to present algebra as a problem-solving tool. We designed our teaching around three central teaching principles, each of which will be addressed below.

3.2. The three teaching principles

3.2.1. Engage students as active and capable learners

The teaching team aimed to cultivate active engagement and agency among the students, which meant that we invited students to bring their own thoughts and ideas into mathematical work. This was a strong principle for the team as various research studies have demonstrated that when students think their job is to passively learn methods many of them disengage (Boaler & Greeno, 2000; Boaler, 2002a,b,c, 2006). In Boaler's different studies of students learning mathematics in contrasting teaching environments she has found that even many of the students who are successful through passive engagement, frequently plan to stop taking math courses at the earliest opportunity (Boaler, 2009).

An equally strong principle among the team was the pursuit of equity. Research studies show both that minority students are often denied access to high level math, through discriminatory tracking and course placement (Boaler, 2016; LCCR, 2013) and that one of the most damaging ideas in math education, held by some teachers and students, is the idea that only some students can be good at math (Boaler, 2013a, 2016). We communicated high expectations for all students both through our teaching design and our teaching practice. We grouped the students heterogeneously so that we could invite all students into rich conversations and so that all students could be working on high-level mathematical work at all times. Heterogeneous grouping, when taught using strategies and tasks that engage students at different levels (Cohen, Lotan, Scarloss, & Arellano, 1999) has been shown in various research studies to be highly effective (e.g., Boaler, 2008a,b; Boaler & Staples, 2008; Burris,

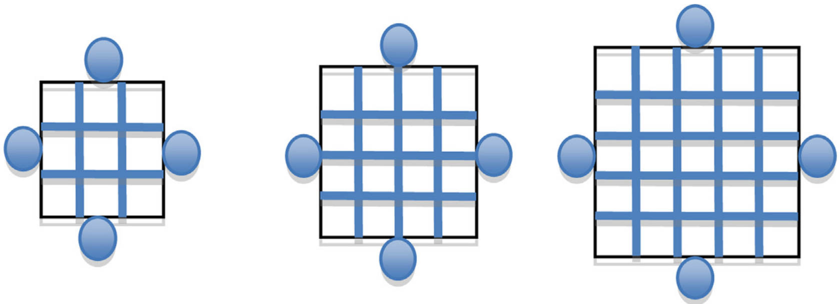

Fig. 1. Example patterning task. Draw or describe the 5th case. What about the 10th case? The 100th case? The *n*th case? Consider starting with Case 1!

Heubert, & Levin, 2006). We also knew that teachers are often challenged by classes in which students have very different mathematical backgrounds, enthusiasm, and achievements and we wanted to use the study to provide insights into the methods teachers may use to address such student differences.

Many textbooks in the US emphasize procedural and low-level conceptual work that make it difficult for teachers to provide opportunities for students to be innovative, create their own mathematical ideas, or extend established ideas (Boaler, 2009, 2016). We chose to use curricula that encourage students' active engagement with mathematics drawing from the Interactive Mathematics Program (Fendel, Resek, Alper, & Fraser, 1997), College Preparatory Mathematics (Sallee, Kysh, Kasimatis, & Hoey, 2000), Parker's (2009) MEC Pattern materials, and two schemes from England – Graded Assessment in Mathematics (GAIM) and SMILE mathematics. Additionally, all the teaching tasks involved materials or resources that were readily available at the school (e.g., Algebra Tiles, Unifix cubes, etc.).[1]

To prepare for the breadth of prior achievement of our students we deliberately chose tasks that we think of as having a low floor and high ceiling. These are tasks that are easily accessed by students with varying backgrounds, that can be seen and solved in different ways and that can be taken to high levels. For examples of these tasks see www.youcubed.org/tasks. Most of the tasks were also extended and took time to solve – challenging the common belief that math problems should be answered in a few minutes (Schoenfeld, 1989). In our summer classes, it was not unusual for students to spend an hour on a complex problem, or to work on a problem over 2 class periods. We also frequently encouraged students to extend tasks they had completed or use alternative methods and solution pathways. This invitation was at first taken up by students with high prior achievement but as time went on, students who had been unsuccessful in their prior math classes also took up the invitation.

Many of our problems used or encouraged multiple representations. An example of one of the problems that we used is *The Border Problem*, which is described in depth by a teacher who used this problem – with associated lesson plans and reflections (Boaler & Humphreys, 2005). Another is the classic *Handshake Problem* ("How many different handshakes are possible among *n* people?"), which elicited multiple representations – tables, tree-diagrams, written explanations and images connecting the vertices of triangles, squares, and pentagons to illustrate a pattern.

There were days when all students were engaged in a common task and other days when students were introduced to a 'menu of tasks' (Parker, 2009). On the menu days, students were given brief introductions to several tasks that they could choose between and that were placed in "stations" around the room. Students could then independently (or with peers) select a task, choose how long they needed to stay with that task or determine if they wanted to extend the task, before moving onto a different "station". On some days we gave three to four problems that were conceptually similar but different in complexity or representation.

Class sessions often began with *number talks* (Boaler, 2009; Humphreys & Parker, 2015; Parrish, 2010) – a pedagogical strategy that involves a teacher presenting a problem to a class, for example 60% of 80, and then asking students time to consider its solution alone, and raise a 'quiet thumb' when they are finished. Students then share how they solved the problem and the teacher records the various mathematical routes to a solution and engages students in discussion of the different methods. Typically, each class would generate 6–8 different methods to a correct solution. Number talks showed students that mathematical problems, even computations, could be represented and solved in different ways. Several lessons were designed around algebraic patterning, an example of which is provided in Fig. 1.

3.2.2. Teach algebraic content through mathematical practices

In 2003, I was on a RAND panel of mathematicians and mathematics educators, led by Deborah Ball, that was tasked with the job of working out productive research agendas for the future, and reporting these to the White House. The group highlighted the importance of learning more about the teaching of 'mathematical practices', now enshrined in the Common Core, pointing out their connections with the pursuit of equity. Practices are the important activities in which successful mathematical problem solvers engage; they go beyond mathematics knowledge to the things mathematicians "do". Examples

[1] The curriculum is available to download at http://www.youcubed.org.

included exploring, orienting, representing, generalizing, and using mathematical language, symbols, and conventions. The RAND group also suggested that when students do not achieve well in mathematics classrooms it is often because they have not been taught to engage in mathematical practices. The RAND group highlighted mathematical practices as an issue of equity arguing that successful students somehow learn these and others do not but when students are explicitly given opportunities to learn practices then previously unsuccessful students may gain success. This was demonstrated by a science intervention studied by White and Frederiksen (1998).

'Mathematical Practices' are now central to the Common Core State Standards (CCSS) adopted in 45 states across the United States, and these practices are intended to be combined with all levels and domains of mathematics content (2010). The CCSS Initiative, sponsored by the National Governors Association and the Council of Chief State School Officers, seeks to bring national alignment in standards-based instruction in Mathematics and Language Arts across participating states. Because of the onset of the Common Core teachers across the US are now required to engage students in mathematical practices but many have little experience of doing so. Our summer course provides a case of curricular design in which algebraic content was taught through mathematical practices in line with the Common Core State Standards.

Our choice to re-present the discipline of mathematics as engagement with content and practices began by the selection of practices that we saw as critical to mathematical engagement. These included, but were not limited to, questioning, representing, generalizing, reasoning and justifying (Maher & Martino, 1996; Martino & Maher, 1999). As we unpacked these practices and considered what they involved, we began to identify pre-cursor activities that are not always explicitly taught or made visible to students but that we believed were necessary. These were, for example: organization, discerning relevant information, choosing to work with a smaller case, and finding systematic ways to track inquiry.

3.2.3. Encourage a collaborative community

Another central goal of our teaching was to encourage mathematical discussions that would increase student understanding and help students feel like members of a mathematical community. A highly consistent area of education research shows the value of collaborative learning (Angier & Povey, 1999; Boaler, 2009; Bransford, Brown, & Cocking, 2000; Cohen, 1994; Engle & Conant, 2002; Leikin & Zaslavsky, 1997; Webb & Palinscar, 1996). In mathematics, students report that discussing ideas gives access to understanding and makes the discipline more interesting and engaging (Boaler, 2008a,b; Boaler & Staples, 2008; Brodie, 2010; Yackel, Cobb, & Wood, 1991Yackel, Cobb & Wood, 1991). Mathematics classrooms that have been shown to create more equitable opportunities for learning are those in which a range of student participation is supported, students are encouraged to ask questions and share ideas, and to feel safe in doing so, and teachers express a commitment to students' academic and social development (e.g., Boaler & Greeno, 2000; Boaler, 2005, 2009; Gresalfi & Cobb, 2006; Gutstein & Peterson, 2005; Hand, 2012; Schoenfeld, 1992). Whether developing disciplinary dispositions (Gresalfi & Cobb, 2006; Gresalfi, Martin, Hand, & Greeno, 2009) or encouraging students to take up space (Hand, 2012) or promoting relational equity (Boaler, 2008a,b; Boaler & Staples, 2008), research in mathematics education inextricably links opportunities for equity to the creation of supportive communities of math learners in which teachers believe all students can learn to high levels (Boaler, 2013a, 2016).

The teaching team was highly committed to students working collaboratively – whether in pairs, triads, or most often, in groups of four. In anticipation of possible challenges, the teachers made several deliberate decisions. First, groups were arranged without any regard to prior achievement and instead were organized around their common mathematical interest in a task or problem set. Second, the first four lessons in the summer had specific activities designed to communicate high expectations for all, engender respect for peers who shared ideas, encourage a focus on listening as much as talking or writing, and communicate the view that different methods and perspectives are important in mathematics. For example, on the first day students were asked to pair up and answer questions about groupwork: e.g., "What helps you when you are working in a group?" "What doesn't help you" and "What do you not want to hear from other students?" Students then shared their responses with the class, which were recorded and posted on the wall. The list, which included: students wanting time to think, not wanting to be left behind, and not wanting their ideas to be ignored, was treated as a working document and contract that was periodically referenced and updated over the five weeks. On the second day, while students rotated through different "stations" in the room, the teachers recorded specific comments or conversations they overheard that represented positive talk and encouraged these. In highlighting students' words the teachers communicated that they were taking the group interactions seriously.

We asked students to share their ideas, encouraging those who were reticent to do so, and we encouraged students to listen to each other and to be respectful of each other's ideas. Students worked in groups or pairs for the majority of the time and we encouraged their discussions with each other and sometimes as a whole class with the teacher facilitating. Sometimes we arranged the groups explicitly to ensure more heterogeneity within groups and sometimes we allowed students to choose their own pairs or small groups. The classrooms were rarely quiet as students discussed the meaning of tasks, argued the merits of particular methods, and deliberated over possible solutions. One of the reasons we taught mathematics through discussions was because we valued both mathematical reasoning and sense making. We encouraged students to reason about and justify their choice of methods, as we knew this to be an important part of mathematical work that requires discussion (Boaler, 2013b). By emphasizing collaboration, students also became accountable for their mathematical ideas and methods in partnership with others.

To encourage the students own voices and ideas we gave all students on the first day of class their own blank journal and encouraged them to write down ideas, mathematical work and questions they might have in their journal throughout

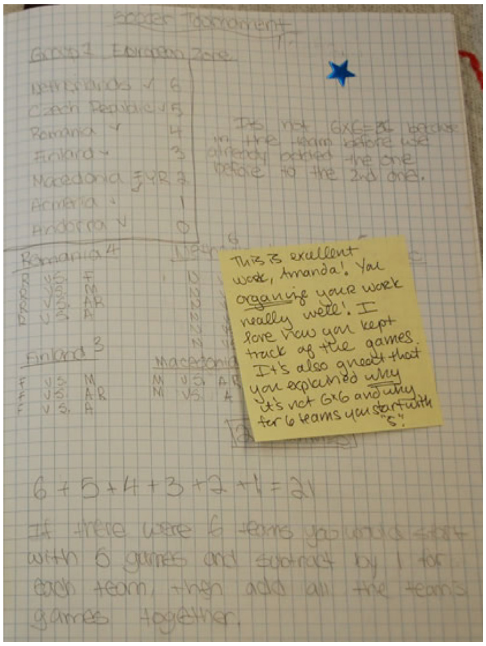

Fig. 2. Student journal entry for a combinatorics problem; teacher's feedback post-it: "This is excellent work, Amanda! You organize your work really well! I love how you kept track of the games. It's also great that you explained why it's not 6 × 6 and why for 6 teams you start with '5'!".

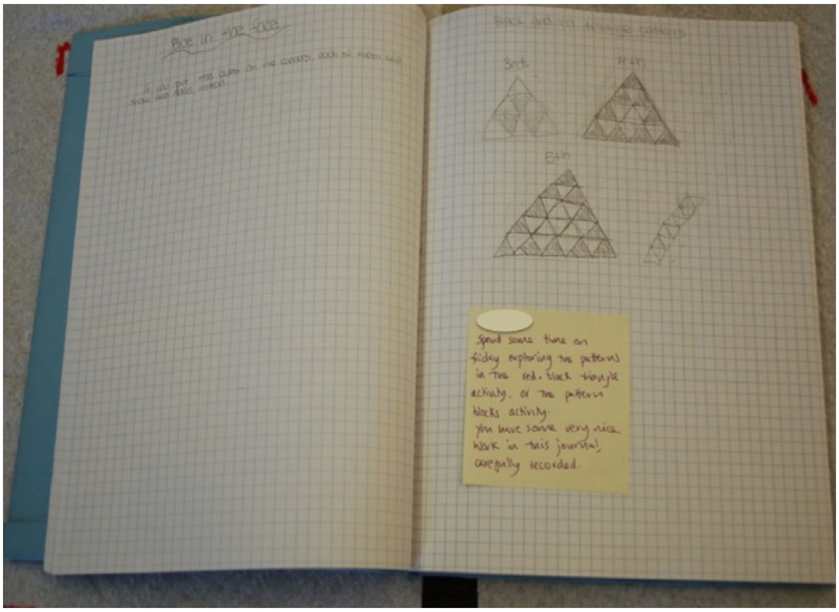

Fig. 3. Student journal entry for patterning problem; teacher's feedback post-it: "Spend some time on Friday exploring the patterns in the red + black triangle activity or the pattern blocks activity. You have some very nice work in this journal, carefully recorded."

the summer. We chose to have students work in journals, drawing from Lampert's teaching (2001) in order that we could value and appreciate their ongoing mathematical thinking. Journals offered students who were hesitant to share their ideas publicly a "safer" opportunity to communicate their thinking (Clarke, Waywood, & Stephens, 1993) and gave the students opportunities to develop and feel proud of their own record of mathematical work. The journals were designed to provide a running record of ideas, of both failed and successful attempts at tasks, as well as trace students' growing proficiency with mathematical practices. We collected the journals at frequent intervals, and provided written feedback on post-it notes. We wrote on post-it notes rather than directly into the journals as we wanted to maintain the journals as students' spaces to write, rather than the teachers.

On the first day of class we encouraged students to decorate and personalize their journals, in order to help them feel ownership for them. Figs. 2 and 3 show students' records of their mathematical work and the teachers' responses.

In addition to the opportunities for students to share their voice through written work, we frequently asked students to present their ideas on the board during their mathematical explorations. The students were not used to presenting their ideas publicly and initially they took come encouragement to do this. Anticipating this we deliberately built in low risk opportunities for students to present ideas at the beginning of the summer. For example, one of the activities we gave students was the task *Four Fours*. In this task students were asked if they could find every number between 1 and 20 using exactly four 4's and any operation. All of the students could find quite a few of the numbers and use creative thinking in doing so. After the students had worked for a while we asked them to come to the board and put up any of the solutions. At this point the board became crowded as students jostled to put up a solution, and everybody seemed happy and relaxed in doing so. As time went on we asked students to come and present ideas at the board in small groups, and sometimes alone, but only when they had volunteered to do so.

4. Conclusion

In this first paper of our collection, I have described the teaching intervention that became our summer school algebra class. The class was accompanied by a research study that explored the impact of the teaching upon students' engagement with and learning of mathematics. In the papers that follow we will present the results of this study with analyses of the students' learning and achievement, and focused analyses of the ways students learned to collaborate, work with agency and engage in mathematical practices. Many of the students in our summer school classes transformed their relationship with math during the summer, moving from initial disaffection and low achievement to engagement, excitement and high achievement. The papers that follow analyze the ways in which this transformation happened, the first paper presenting the research evidence for this change and the later papers providing in depth analyses of the students engagement with mathematical practices and each other.

Acknowledgement

This research was funded under a grant from Teachers for a New Era.

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