In December 2016, Jo Boaler interviewed Steve Strogatz at Cornell University and observed his Math Explorations class for undergraduates. Below are selected excerpts from that interview. (JB = Jo Boaler, SS: Steve Strogatz)

JB: I am really interested in what you would say to the question of what is mathematics really.
SS: It [mathematics] has competitive aspects, cooperative aspects. It's very social. This is something that many people don't think. They would imagine that it's a solitary activity. And sometimes it is, we'll spend time thinking by ourselves.

But there's nothing better than being stuck on a problem and talking to another person who's interested in math and wrestling with it together, and then sometimes managing to solve it together. So there's the social aspect.

It is about convincing. It's a social act. I mean, you can prove things to yourself. So I suppose you could argue it doesn't have to be social. It's just you versus logic. But in reality, it tends to be a social act, where you're trying to make a persuasive argument to convince others and to help them understand why something is true.

…in math, we're very concerned with proof- but why? Is it just to be sure that we're right? That's part of it. But part of it is to understand why something is true. That as many mathematicians would say, that the key thing we're looking for in a proof is insight, not just to be sure that something is true but to understand in our gut why does it have to be true.

Math has this profound connection, a very mysterious connection, to the real world, where we can use mathematics to understand the rhythms within our own bodies, the atmosphere, the ups and downs of the stock market, the course of epidemics, and, of course, physics.

And people will say, math is the science of patterns or the science of numbers. It's related to science, but it's not just a science. Because it has a lot in common with art. That is, there's a very creative dimension to math, including the role of imagination. We can make up mathematical objects by free will and investigate their properties.

Strogatz on developing math insights
SS: But what I loved about the [lesson on paperclips and pennies] was-- and also what the student Simone, who was having a-- well, what should we call it-- I mean, she was having a moment of revelation.

She was like, oh, you know, and I hope you captured on film. She was really like slapping her head. I think the other students we're looking at her. A few of them were having aha moments too. Though I think some were not. But in any case, there was definitely a moment of insight from what Jay did and it was felt by Simone and some of the other students.

Strogatz on Rigor and Creativity
SS: What I would want to underscore about that is that we think of math as a logical, rigorous-- you always hear this word rigor.

I would object to that too. Rigor is only half of what we do in math. Rigor is the over-emphasized part of math at the expense of creativity and ingenuity and intuition. You know, you might say intuition is the almost the opposite of rigor.

And yet without intuition there is no way of even starting the proof. I mean, of course, without rigor, we don't have a proof. So I need both. And all mathematicians I think would agree that our subject combines creative thinking with critical thinking. There's too much talk about critical thinking, or even the way that this subject is named here at Cornell. It's considered part of the mathematical and quantitative reasoning requirement, the MQR requirement, as if reasoning is the only game in town. And it's not the only game in town.

There's creativity that doesn't involve reasoning that involves hunches, and emotional feelings, or even body feelings. Like wouldn't it be nice if-- you know, there's a kinesthetic aspect to mathematical creativity. So, in fact, to exploit that, I have the students do some exercises that are literally exercises, that are standing up and moving, dancing, striking poses. We did that in the recent classes to explore symmetry.

Strogatz on Intuition and Mistakes

SS: The mathematicians I know realize that to make a proof, which is what-- in the game of pure math that is sort of what the activity is mainly focused on, is finding true theorems and proving them. And everyone I know realizes that you have to make a mess first. You're confused, you're groping around in the dark, you don't know where you are. And trial and error is a key technique.

You have to make mistakes. And part of that, in the social realm is that you have to work with people that you trust. Because you're going to be vulnerable and look stupid a lot of the time. And if you don't do that, if you don't take chances and looked foolish, because you have to learn, you won't really do anything very remarkable. You'll be playing it too safe and just inching around from what you already know.

JB: ***That's very interesting. Because that holds students back in class so much that they are scared of ever making a mistake or doing anything wrong.***

Well, we're scared with each other too. Professional mathematicians are scared to look stupid in front of other mathematicians. It's not good for our reputation. But with a trusted collaborator and you're working on a problem together, you have to be prepared to look stupid and to be generous in reaction to your partner's occasional silliness.

There are wonderful examples of that. Poincare, the great mathematician of the late 19th and early 20th century, was trying to solve a problem that had been posed by the King of Sweden, the so-called three body problem in astronomy. And everyone was trying. It was a prize competition. All the best mathematician's in the world are interested in this.

And it have defied people for several hundred years. Newton, himself, had said about 200 years earlier that this problem had made his head ache. No problem had ever made his head ache like

the three body problem. Which has to do with the mutual attraction of the sun, the earth, and the moon, those being the three bodies.

Two bodies, the sun moving around-- or the earth moving around the sun, Newton had solved. But include the moon-- so Newton actually referred to it as the problem of the moon. And it's a very practical problem because it has to do with the tides on the earth and understanding the patterns of the tides relates to the sun and the moon.

Anyway, Newton couldn't solve it. Fast forward 200 years, Poincare is trying to solve it. And he can't really solve it until he has a brilliant idea. And that he thinks he has solved it. And he even writes the paper that then wins the competition, except it turns out it's wrong. And you mention about the generative aspect of mistakes.

This was one of the most important mistakes in the history of mathematics. Poincare's mis-solution of the three body problem. And when he realized his mistake, actually a referee to his work pointed out that there was something a gap in the argument, he looked at the gap and realized he couldn't quite patch the gap, that there was a serious problem there that was not easily fixed.

He had to stop. He was so embarrassed that he made all the copies of the prize winning manuscript be destroyed. So that no one would ever know about his mistake. In fact, we know about this because one of them was not destroyed. So we've seen Poincare's original interesting wrong solution.

But in any case, he later did solve it correctly by inventing chaos theory. So in terms of generative mistakes, this was the mistake that led to the birth of modern chaos theory. And without that mistake, who knows.

Strogazt on Seeing Math as Creative

SS: It's shocking to me that people don't see math as creative, though I understand why not. The way we teach it, tend to teach it in elementary school, K to 12, and even in college does not tend to emphasize creativity, to put it mildly. That is, you're taught that you should follow the rules, you must use precise definitions.

In fact, almost all the wrong messages are given, because even coming up with a definition is a creative act, or choosing what to define is a creative act. You know, it is incredible how much-- how wrong-headed the picture is that we give of mathematics.

Just figuring out what you're trying to define, or what does it mean to speak of instantaneous velocity-- if something is moving in a way that isn't at constant speed, but is speeding up. Your speedometer on your car registers a speed at every moment. So there is an instantaneous velocity. But how is that being calculated? That was a problem the bothered Fermat, and Descartes, and even Newton when calculus was being invented. And figuring out the right definition took a lot of work. Nowadays, we just present the definition of a derivative or of

instantaneous velocity as if, ho hum, this is the obvious definition, concealing all the struggle in the creative energy and imagination that had to go into finding the right definition.

The theme of creativity brings to mind other dimensions of mathematical activity in that the questions we ask embody creativity also. That is no one tells a mathematician what questions to ask, or what to wonder about, or what to explore. Someone has to first come up with a puzzle or a mystery. And that choice of where do I focus my attention or my curiosity is itself a creative act. And this is something that every person of every age can participate in.

If you ask that question[about the M&M task], you've already asked a creative-- you've made a creative challenge because you now realize that this little activity has a mathematical problem in it.

Strogatz on Math Teachers and Meaningful Math
SS: Many teachers know that it's very effective if the student is genuinely interested in the question. Students like to figure out the answers to their own questions. But to a question that comes from someone else, especially a question that feels meaningless, we spend so much time making them answer questions that mean nothing to them.

I see it with my own children, who are now 14 and 16. But I especially remember at an earlier time in their math education that they would be given lots of exercises that they could perform, and really mean perform, almost like a trained animal, the solutions to without having any idea why do I want to factor a polynomial. I don't really know why I'm interested in that, and I'm not interested in it, but I have 50 of them to do for tomorrow. So I'll just do it.

And if I try to get them interested, very frequently they'll say dad, just-- I have to get 50 of these done by tomorrow, can you just show me what to do.

JB: What do you think is important for students to know about math? What should they know that math is?
SS: For me, there are three things that I would hope people could take away. And not everyone has to take away all three.

So those kinds of things, also for understanding the problems of our time, thinking about climate change reasonably, understanding the risk of having a gun in the family versus having a swimming pool, or there's all kinds of things that people don't tend to have a good grasp on unless they have some knowledge of probability, and statistics, and number sense.

And we do teach arithmetic about how to add, subtract, multiply, and divide. And of course, we do fractions and decimals after that. But I wish we would spend a little more time about how all these arithmetic ideas matter in your real life as a taxpayer, as a person who goes shopping, as a person who has a bank account.

OK, so you could call that like citizen numeracy or something like that.

There's also the math that you would need to be in the most exciting high paying jobs of the century. Now, not everyone wants those jobs. But if you want to work for a hedge fund and get really rich, or go into a super exciting part of radiation oncology and help cure cancer or treat cancers-- I mean think about all the interesting professions in tech, and finance, and pharmaceuticals. And we could go on and on.

But when people say that you need quantitative skills for the 21st century, it is true that a lot of the great jobs do require much more than arithmetic. So computer literacy, and comfort, and a lot of the things that we currently teach like algebra, and geometry, and trigonometry, and calculus, it would be great if people studied those, and mastered them, and enjoyed them, and more.

Because I would like people to learn about networks and probability. There's a lot going on in the math of the 21st century that is very practical for jobs, and for understanding the world around us, and inside our own bodies, many of which we're not teaching. So I would love to see people learn much more math. But not everyone-- OK, so, to argue against myself for a second-- not everyone wants to become a hedge fund manager, or an oncologist, or a person working at a big data company.

There's the third part that I think I want everyone to know, and this may be me being crazy. But just like you need to know how to read, but you would benefit from reading Shakespeare, and Toni Morrison, and EE Cummings, I want people to read the poets of mathematics and appreciate how beautiful and world changing the work of the great mathematicians was.

That is math is part of our culture and our civilization. And it's not just about you learning the skills for your job or to pay your taxes. There's also you being part of the tradition and part of the conversation. So for me, calculus, and graph theory, and probability, these are great triumphs of the human imagination and spirit.

And just like we have introduction to music, kids learn about Mozart and Beethoven, not because they're going to become professional musicians in every case, but because it makes your life richer to know about Beethoven or the Beatles. It doesn't have to be lofty stuff. Can't we do that with introduction to math? Why isn't there a Math 101, like we have Psych 101? Everyone wants to take Psych 101 and learn about the quirks of your brain and of social life.

Strogatz on the need for a Math Revolution

SS: Because it [math] is invisible all around us, but it's made our life modern. And so by spending so much time on just developing the ability to factor polynomials or to know the difference between dependent and independent random variables-- like we spend so much time on minutiae, that kids don't-- I mean, you need that. You need that if you're going to become a rocket scientist, but most people aren't.

This is what makes me crazy. That we really do a bad job of serving 90% of the students. I'm making up that number, but, I mean, a large number-- think about how many people hate mathematics.

There's a reason they hate it. And the reason they think it's worthless. And their parents say I was never good at math and you don't need to be either. There's a whole systemic problem. This is much bigger than the question of is the Common Core good or not. We really need a revolution.

Strogatz's Ideas for Updating the Math Curriculum

SS: I think we assign generally too much homework. I think it would be nice to have less homework and more time to think, and maybe different kinds of homework where students could write reflections, ask their own questions, keep journals. I mean, I just think it's a little one dimensional right now. And it's stultifying.

I do think what we have now looks like it's geared towards an era when it was designed. I'm not sure this is historically correct, but it looks to me like if I wanted to design a curriculum for rocket scientists working for NASA to put someone on the moon, it would look like what we have. That it's a build to calculus through algebra, geometry, and trigonometry, and vectors.

That's a really good design if your goal is to make trajectory engineers, people who could design rocket trajectories, use laws of physics, and astronomy, and so on. And that is what we once did. You know, when you think about the effect that Sputnik had on US education around 1960, I think a lot of the curriculum was baked in at that time.

Times have changed a lot. And I don't see much rethinking happening. The Common Core-- I don't have an objection to having standards, but it feels like such a tweak, such as a minor tweak on this rocket science curriculum. So I think if we asked-- let's start over-- what do you want now?

And let's be creative about it. And this, by the way-- we're not talking about dumbing down. I hope people don't think of it as that, because we would be putting in lots of interesting very significant and substantial mathematics and keeps some. Personally, I would keep a lot of calculus because I think-- of course that's not for everyone- but given that some students will be taking it, what I like about calculus is how central it is mathematically, that through calculus you can understand some of the phenomena in trigonometry that otherwise don't make sense or in even parts of geometry.

It's not just that it teaches us about the world. It teaches us about the rest of mathematics too. So I hear people saying lately calculus is old fashioned. Linear algebra is the subject of big data. We should be going more digital and more "big data" so there are people advocating for statistics, probability, linear algebra, data science, and computers. All very important.

I mean, if you're working at Yelp or Google, those are the parts of math you would use, maybe more so than calculus, I think. And differential equations would look kind of like that's for the

engineers, that's not what a data scientists needs. I would hate to see the calculus and the continuous parts of applied mathematics and mathematics be lost in the digital revolution.

So I kind of-- back to where I started-- I want all of it, including the history, by the way. I personally think we should be teaching the history of the subject. And also the connections to pop culture, to music, to art, to sports-- it's everywhere-- biology, medicine, law, why not?

JB: Why do you teach the Math Explorations class?
SS: After 16 years of being told what to do mathematically and just doing it. Here's your chance to explore and ask your own questions. I mean, we have guided activities. We're not just letting people do anything they want. But within the framework of these guided activities, or explorations, or investigations, of whatever you want to call them, there's a lot of room for imagination and free thinking.

And many of the students have never experienced this before. And it's our attempt to show them the creative and imaginative dimension of math and the breadth. I mean, we do connect-- so you could call it math for liberal arts.

And we tie it into reading. I have them keep journals. I have them record their frustrations. We talk about emotional things. They write their mathematical autobiography. Tell us about a teacher that inspired you or that frustrated you, or traumatic experiences, and the like.
The main goal is to change their attitude, to help them see how exciting and interesting this subject can be, and how worthwhile it is, and how enriching it is. So it's about a positive attitude change. That's what we're hoping for.

My job is not to teach them any particular thing. It's to show them what it feels like to actually do mathematics and why anyone would want to do that. So I don't have the constraints that a teacher who has to bring her students up to the level of passing a certain test mandated by someone else. That person is under a lot of pressure that I'm not under. So when you ask could I teach all of math this way, it would be a real challenge if there was someone else telling me what my students need to know by a certain date.

Strogazt on Managing Math Discussions
SS: It's hard to teach that way, much harder than lecturing.

I don't want any one student dominating. I'm looking at the faces of every student to see how engaged they are, how bored they are, how lost they look or not. And I want them to be at the edge of what they can do. That is it should be interesting enough to be stimulating and really engaging, but not so hard that they're just stumped and sitting there.

I mean it's OK to be stumped. That's a normal mathematical experience. But it should be clear what the question is. We spend a lot of time trying to figure out what is it we're doing. And I'll have that actively discussed, what are we doing, what are we trying to prove.

And I want it to be coming from them. I try to be as quiet-- I'm not doing well with our interview here, but I try to be quiet as much as I can. And so I hope that when you observed you saw I wasn't really-- I tried really hard not to lecture. I want to just pose the question or an activity, let them do it. I also try not to judge their answer explicitly.

If they answer and it may be quite muddled what they're saying, because when you're confused, it's going to come out muddled. Instead of trying to correct them about the words they're using or the ideas, I try to be as neutral as I can. I might just record what they said.

There was a moment towards the end of the class where I felt like a payoff was coming, that one student had brought us very close to the solution of this thing that had been bothering the class for at that point maybe 30 minutes, or 45 minutes, and I wanted them to experience the aha moment.

And, of course, you could just be completely neutral and let anything happen that's going to happen, but I like to sort of steer it a little bit. I think that's OK. I think it's desirable. And there was another student who looked like he was going to take us in a different direction. And it's a hard call. Do you respect that and let it go in that direction? And normally, I want to.
But I really felt we were on the brink of a big moment. And so I kind of tried to calmly, and not too obnoxiously-- I want to say cut him off, but redirect him or-- so I don't know how successful that was. I'll be interested to look at it on the tape.

Strogazt on Future Directions for Math Education
SS: What I like to see happening? I really wish that students could have the experience of doing math as it really is. That's what I miss and what I worry about, that they're seeing-- maybe Paul Lockhart put it this way in his mathematicians lament-- that the problem with math class is that there's no math in it.

You know, if you think about that take that seriously, that there's this dog work of just grinding out mindless examples on these stupid work sheets. That is not math. I mean, that's a part of math. That's like, that is to math what playing scales is to music. You have to play scales to become a competent musician. But that's not music people go into music because of our beautiful it is because of the pleasure it gives them because of the challenge of actually doing it, not just reading sheet music, but actually playing music badly and getting better by hearing your mistakes, and having a gentle teacher guide you and reward you and all that.

And plus the reward of yourself-- you hear its bad or getting better. That can all happen with math. You have to let people play math like they play music or play soccer. We don't allow for the most part the actual playing of math because we're so constrained by these ridiculous rules that are in place, by people who are well-intentioned

Strogatz on Attributes of Math Learners
SS: We don't reward that impulse to ask that question or make that connection. That's a pity, because that's very important at the highest levels of creative work, at the research frontier. I've been rewarded for being someone who likes to do that and has done that, making connections.

And in terms of being the fastest kid and the most powerful logical brain, I was always near the bottom of my class. I really was. I mean, if I had let that stop me, I would have stopped.

Every student that was a math major in my college-- I would say was better than I was by the standard measures. But they didn't necessarily perform better on the tests because I really loved the subject and I thought about it a lot. And even if I was discouraged by being slow, which I am, and my wife likes to tease me. But I just am-- I'm slow. So what. Many mathematicians are slow and real math is not about speed.

It's about thinking, and loving, and pursuing the subject, and communicating it too. That has been a helpful thing for me-- trying to spread the joy, and the pleasures of math, and the power to solve important problems in our real life and in medicine and other subjects. That counts too.

I mean, math is not a disembodied separate part of our culture. There's part of it that acts like, the most pure of the pure math. But, you know, there's this whole other thing-- applied math. That's the world I live in-- math applied to everything. And that's very rich and interesting and it's also part of the mathematical universe.