Extract from *Mathematical Mindsets: Unleashing Students' Potential Through Creative Math, Inspiring Messages and Innovative Teaching* (Wiley, 2015 pages 190-191) by Jo Boaler.

Mathematics is a subject that should be highlighting depth of thinking and relationships at all times. In a recent visit to China I was able to watch a number of middle and high school math lessons in different schools. China outperforms the rest of the world on PISA and other tests, by a considerable margin (PISA, 2012). This leads people to think that math lessons in China are focused upon speed and drill. But my classroom observations revealed something very different. In every lesson I saw teachers and students worked on no more than three questions across an hour's lesson. The teachers taught ideas, even ideas that are among the more definitional and formulaic in mathematics, such as the definitions of complementary and supplementary angles, through an inquiry orientation. In one lesson, an extract of which can be viewed here: www.youcubed.org/high-quality-teaching-examples/, the teacher explored the meaning of complementary and supplementary angles with students, by giving an example and asking students to "ponder the question carefully" and then discuss questions and ideas that came up. The ensuing discussion around complementary and supplementary angles moved into a depth of terrain I have never before seen in my observations of mathematics classrooms teaching this topic. The teacher provocatively took the students' ideas and made incorrect statements for the students to challenge and the class considered together all of the possible relationships of angles that preserve the definitions. This is an extract from a typical US lesson on complementary and supplementary angles, taken from the TIMSS video study of teaching in different countries (Stigler & Hiebert, 1999):

US example, from http://www.timssvideo.com/

> **Teacher:** Here we have vertical angles and supplementary angles. Angle A is vertical to which angle?
>
> **Students chorus**: 70 degrees
>
> **Teacher:** Therefore angle A must be?
>
> **Students chorus**: 70 degrees.
>
> **Teacher:** Now you have supplementary angles.
>
> What angle is supplementary to angle A?
>
> **Students Chorus**: B
>
> **Teacher:** B is and so is…?
>
> **Students:** C
>
> **Teacher:** Supplementary angles add up to what number?
>
> **Students:** 180 degrees

In the extract above we observe definitional questions with one answer, that the teacher is leading students towards. Compare this with one of the lessons we watched in China, in which the teacher did not ask questions such as "Supplementary angles add up to what number?" She asked questions such as: Can two acute angles be supplementary angles? Can a pair of supplementary angles be acute angles? These are questions that require students to think more deeply about definitions and relationships. Here is an extract from the lesson in China that I watched and that stands as an important contrast to the US lesson.

> **Student:** As he just said if there are two equal angles, whose measures add up to 180 degrees, they must be two right angles. Because the measures of acute angles are always smaller than 90 degrees, the sum of the measures of two acute angles will not be larger than 180 degrees.
>
> **Teacher:** Therefore, if two angles are supplementary, they must be two obtuse angles?
>
> **Student:** That is not correct.
>
> **Teacher**: No? Why?
>
> **Teacher**: I think if two angles are supplementary, they must be two obtuse angles.
>
> **Student:** I think they could be an acute angle and an obtuse angle.
>
> **Teacher:** She says, although they cannot both be acute angles, they can be one acute angle and one obtuse angle.
>
> **Student:** For example, just like the Angle 1 and Angle 5 in that question. One angle is an acute angle. The other one is an obtuse angle.
>
> **Teacher:** OK. If two angles are supplementary, they must be one acute angle and one obtuse angle?
>
> **Student:** That's still not accurate.
>
> **Student:** You should say, if two angles are supplementary, at least one of them is an acute angle.
>
> **Other Students:** No, at least one angle is larger than 90 degrees.

Student: An exception is when the two angles are right angles.

The lessons from the US and from China could not have been more different. In the US lesson the teacher fired procedural questions at the students and they responded with the single possible answer. The teacher asked questions that could have come straight from books, that highlighted an easy example of the angle, and students responded with definitions they had learned. In the lesson from China the teacher did not ask pre-decided questions, she listened to students' ideas and made provocative statements in relation to their ideas that pushed upon their understanding. Her statements caused the students to respond with conjectures and reasons, thinking about the relationships between different angles.

The second half of the lesson focused on the different diagrams students could draw that would illustrate and maintain the angle relationships they had discussed. This involved the students producing different visual diagrams, flipping and rotating rays and triangle sides. Students discussed ideas with each other and the teacher, asking questions around the ideas, pushing them to a breadth and depth I had not imagined before seeing the lesson. As the class discussed the visual diagrams of angle relationships one of the students reflected: "This is fascinating". There are not many students who would have drawn this conclusion from the US version of the lesson.