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FROM PSYCHOLOGICAL IMPRISONMENT TO INTELLECTUAL FREEDOM – THE DIFFERENT ROLES THAT SCHOOL MATHEMATICS CAN TAKE IN STUDENT'S LIVES.

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Some years ago I studied two schools in England that taught mathematics very differently. The contrasting approaches resulted in students developing different ways of knowing mathematics. In a follow-up study I investigated the students' knowledge-use in life some eight years after leaving school. This showed that the two mathematics teaching approaches had caused profound differences in the ways that the young adults interacted with knowledge. In this presentation I consider these differences in terms of adaptive expertise (Hatano & Oura, 2003) and mathematical identities (Wenger, 1998). The two studies – of the students in school and of their knowledge use as adults – offer the opportunity to examine the different ways that people may hold, use and relate to mathematics knowledge, in school and beyond.
Teaching, mathematical identity, adaptive expertise

INTRODUCTION

As a researcher in England, I studied two different approaches to teaching mathematics and looked at their impact on student learning. I followed cohorts of students in two schools, over three years. The different cohorts were similar in terms of social class and prior achievement and had experienced the same mathematics approaches up to the age of thirteen. At that point their pathways diverged with the two groups attending schools with very different approaches (Boaler, 1997a, 2002a). I followed students through their mathematics classes from age 13 to 16, collecting a range of qualitative and quantitative data. These included hundreds of hours of classroom observations, interviews, questionnaires and assessments. 'Amber Hill' was a comprehensive school that taught mathematics in a fairly traditional way. Students were placed into one of eight ability groups for mathematics at age 13. They were taught using textbooks, teacher lectures, and practice. 'Phoenix Park' was an unusually progressive school. Mathematics was taught in mixed ability groups until the latest possible moment – a few months before the national examinations, and students worked on open-ended projects in class. The teachers at Phoenix Park would introduce a few projects to students who would then choose what they wanted to work on. They would only be taught new mathematical methods when they needed them to solve problems. For more details see Boaler (2002a).

One important outcome of this longitudinal study was the high achievement of the students at Phoenix Park. They scored at significantly higher achievement levels than the Amber Hill students, on a range of assessments. These included the national examination, despite being at the same levels at age 13. They also scored at higher levels than the national average, despite being at lower levels when they entered Phoenix Park. One of the reasons that the Phoenix Park students outperformed the Amber Hill students was the open-ended mathematics approach they experienced and the higher levels of interest they developed in mathematics through this approach. Another reason was the ability grouping at Amber Hill. Although I did not set out to study the impact of ability grouping, it emerged as a critical factor in the students' achievement. Many of the students reported that they gave up on their learning when they were placed into any group from 'set 2' downwards. At Phoenix Park the teachers grouped the students as late as possible. Some initially low achieving students, who would have been placed into a low group had they been at Amber Hill, accelerated in their time at Phoenix Park and ended up gaining high grades in the national examination.

Another important finding from the study was the equitable nature of the Phoenix Park approach – there were no achievement differences by gender, ethnicity, or social class, an unusual and important achievement for a school. At Amber Hill typical patterns emerged. There was a significant correlation between the social class of students and the group into which they were placed (r=0.25) after controlling for achievement. Investigation of the students who scored at higher or lower levels on the national examination (GCSE) than might be expected from initial achievement showed that most of the high achievers at Amber Hill were middle class and most of the low achievers were working class (Boaler, 1997a,b).

SCHOOL MATHEMATICS AND LIFE

When reporting the results of this study, I have frequently been asked about the future directions and achievements of the students who attended Amber Hill and Phoenix Park. Some eight years later I received the opportunity to investigate this question by conducting a follow up study of the students. The most difficult part of the follow-up study was the challenge of finding the young adults, who were then around 24 years old. The only contact information I had for them was their old addresses, where they had lived while in school. I sent a letter explaining the research and a questionnaire to the entire two cohorts, a total of 288 addresses (181 from Amber Hill and 107 from Phoenix Park). This resulted in 63 returned questionnaires, representing 20% of the students from Amber Hill and 24% of the students from Phoenix Park. This return is small although understandable given that many of the ex-students had moved away from their old addresses. Despite the relatively low return, the sixty-three young adults who responded were an interesting and important group to consider.

From the data I had collected on the students when they were in school, I was able to investigate the representativeness of the group by social class and by GCSE achievement. This showed that the students who responded were highly representative of the whole school cohorts. Comparing the group who replied at Amber Hill to those who did not, and the same at Phoenix Park, there were no significant differences in social class (Amber Hill: ks = 0.0948, p-value = 0.9387; Phoenix Park: ks = 0.1746, p-value = 0.607). Comparing GCSE scores, the group who responded at Phoenix Park was not significantly different from the group who did not (ks = 0.0683, p-value = 0.9994). At Amber Hill the GCSE comparison showed a difference with the group who responded having significantly higher scores than the group who did not (ks = 0.2575, p-value = 0.0171). This contributes to the fact that a comparison of GCSE scores of those who responded from Phoenix Park with those who responded from Amber Hill showed no significant differences between the two groups (t = -0.8464, df = 58.2, p-value = 0.4008). Even though the Amber Hill students did not score as highly as the Phoenix Park students in school, the group who replied were above average for the school and so were comparable to the Phoenix Park group, who were in turn representative of their whole school cohort. These statistics mean that the 63 students were broadly comparable to each other, and to the bigger cohorts, with the Amber Hill students being somewhat higher in achievement than their whole cohort.

Given the comparability between the two groups, it was interesting to find that the Phoenix Park adults were working in jobs that were significantly higher in terms of social class, than the Amber Hill adults (t = 2.09, d.f. 63.00, p = 0.04.). In the questionnaires the two sets of ex-students named and described their current employment. I categorized the young adults' jobs by social class, using the Office of Population Censuses and Surveys classification (OPCS, 1990a, b, c), the same classification scheme that I had used to analyse their parents' jobs when the students were in school. The jobs were then categorised again, by a second researcher, giving an inter-rater reliability of 88%. The disputed categories were reviewed and agreed, giving the results in table 1 below.

Table 1. Percentages of ex-students in each social class category, OPCS, 1990.

	Professional	Intermediate	Skilled non manual	Skilled manual	Partly skilled	Unskilled	
	1	2	3	4	5	6	n
PP	0	44	30	15	11	0	27
AH	0	25	36	17	11	15	36

Categories 1, 2 and 3 are typically regarded as middle class, 4, 5 and 6 as working class.

These show that the social class levels of the Phoenix Park young adults are now significantly higher than their Amber Hill peers (t = 2.09, d.f. 63.00, p = 0.04).When comparing the social class of the ex-students to their parents (table 2), notice that most of the Phoenix Park adults (65%) improved their social class categorization, whereas approximately half of the Amber Hill adults (51%) went down and a further 26% stayed at the same level. At Phoenix Park there was a distinct upward trend in social class among the children. At Amber Hill there was not.

Table 2: Social Class Movement, Percentages of students (places moved up or down the OPCS scale)

	Downward movement	Same	Upward movement	

					level					
	-4	-3	-2	-1	0	+1	+2	+3	+4	*n**
PP	0	0	5	10	20	40	20	0	5	20
AH	6	0	17	29	26	11	9	3	0	35

*These numbers are slightly smaller than those in table 1 as I only had data on the parents' social class for 55 of the 63 respondents.

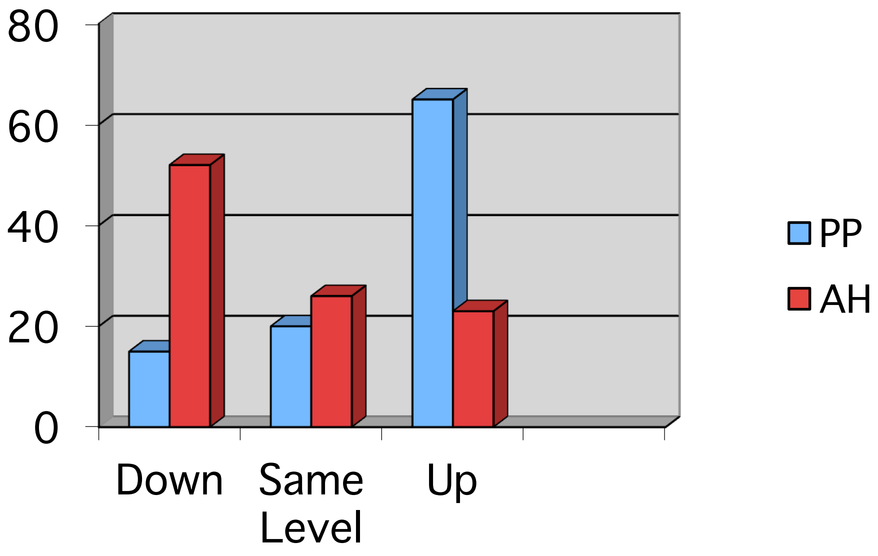

Figure 1. The percentages of students at the two schools who moved down the scale, stayed at the same level, or moved up the scale.

$$\chi^2 = 10.51936 \text{ d.f.}= 2, p=0.005(1)$$

The data above seem to show that the Phoenix Park adults, with their very different mathematical experiences in school, were given a head start in life. And while it is difficult to separate the influence of the students' whole school experiences and their mathematical experiences, the students' whole school experiences varied along similar dimensions. Amber Hill was a traditional school where most subjects were taught traditionally and employed ability grouping, although mathematics divided students into the most groups (eight). Phoenix Park was a progressive school, proud of its tradition of giving students' responsibility and employing project based teaching methods across the school. Only one department (science) employed ability grouping.

One explanation for this interesting result might be the affluence of the two areas and the job opportunities provided in the different locales in which the young adults lived; but this hypothesis cannot be supported by the areas in which the adults lived. Indeed, the Amber Hill adults live in an area that is relatively more affluent with a much wider range of jobs available to them. Phoenix Park school is situated in a more working class area, and most of the students who attended the school lived on the same housing estate (similar to what is called a "project" in the US). The only alternative explanation – that their different school experiences gave the Phoenix Park students a better start in life and afforded them the opportunity to move upward in the social scale – seems likely. Indeed this small but representative data set would suggest that Phoenix Park, a progressive school in one of the poorest areas of the country, helped the students to become upwardly mobile.

I was given further opportunity to investigate the influence of the students' school and mathematical experiences upon their lives by conducting interviews with 20 of the young adults, 10 from each school. These interviews, although they were conducted with only small numbers, showed a marked and interesting difference between the groups. They revealed that the young people from the two schools had developed profoundly different approaches to mathematics knowledge and to knowledge use in life. In considering the most appropriate way to present and understand these differences, I have chosen to draw upon two theoretical perspectives. First, situated theory, which draws attention to forms of engagement, to the practices in which people engage, and to the identities they form through engagement with practices, is helpful in highlighting and combining actions and beliefs (Lave, 1988; Wenger, 1998). Through consideration of students' different engagement with practices and concomitant relationships with knowledge I have come to describe their different positions in terms of their mathematical identities (Boaler, 2002b). The second comes from Giyoo Hatano and his colleagues who conceptualised adaptive expertise (Hatano & Oura, 2003). This lens seems important in highlighting what the young adults knew and could do, which is not mutually exclusive from the situated lens but the different lenses serve to draw different dimensions of the student capabilities into perspective (Lerman, 2000).

To analyse the interview data, all of the transcripts were coded using a system of open coding (Miles & Huberman, 1994). The different themes that emerged were then combined into broader categories. The insights produced by this analysis will be considered in the rest of this paper. I will first present the data obtained from the interviews under three headings: memories of school mathematics, relationships with knowledge, and ability grouping. I will then revisit the findings from the two theoretical perspectives I have indicated, of situated learning and adaptive expertise.

Exploring the Interview Data – Part 1. Memories of School Mathematics.

In interviews with adults from the two schools, the young men and women talked about the ways their mathematics teaching had impacted their lives, giving meaning to the patterns observed in the data on students' social class movement. Although the two sets of adults who responded had scored at comparable GCSE levels, the school experiences of the Phoenix Park adults seemed to have given them important advantages. The Phoenix Park adults reported that their school had excelled at finding and promoting the potential of different students and that their teachers had regarded everyone as a high achiever. The Phoenix Park adults communicated a similarly positive approach to work and life, describing the ways they used the problem solving practices they had been taught in school to get on in life. The Amber Hill adults, by contrast, told me that their ambitions were 'broken' at school and their expectations lowered. They told me that they had been taught to expect little of their own achievement, and most of those I interviewed were unhappy in the jobs they were in, believing that they could have accomplished a lot more.

At the beginning of the one-hour interviews, I asked the young adults what they could remember of mathematics teaching in school. Their descriptions fit extremely well with the ways I remembered and had described their experiences (Boaler, 2002a). For example, the Amber Hill adults, such as Chris, described mathematics lessons in these ways:

> "You just had to basically turn up for your lesson, have your lesson in front of you, "this is what we're covering today". Like, my language class was a similar thing—parrot-fashion learning, "this is what we're doing today". And basic rules to follow."

By contrast, Simon's description is typical of the ways the Phoenix Park adults remembered their classes:

> "It gave you the opportunity to explore things in-depth or to whatever level that you wanted to take them. And I always remember there would be the…you'd work on something for however long it was, and then you would have a discussion with the teacher, and they would perhaps plant another seed in terms of, "Well think about this." And then you'd take that. And it was very much trying to get *you to do the thinking*."

In these different statements, the adults highlight what I also reported as the key difference between the two school approaches. In one the students were taught to follow rules; in the other they were given freedom to explore and to think about mathematics.

The young adults were also very different in their satisfaction with their school approaches. Interestingly, the adults from Amber Hill told me that they now enjoyed mathematics – even though most of them had told me it was very boring when in school. They saw mathematics all around them in their jobs and lives, and many of them solved mathematical puzzles such as Soduku in their spare time. Sadly the adults expressed dismay and confusion that school mathematics had been so uninteresting and unrelated to the mathematics they now saw all around them in life. Sharon represented this view clearly:

> "It was never related to real life, I don't feel. I don't feel it was. And I think it would have been a lot better if I could have seen what I could use this stuff for … because it helps you to know <u>why</u>. You learn <u>why</u> that is that, and <u>why</u> it ends up at that. And I think definitely relating it to real life is important."

Marcos communicated a similar view, highlighting the procedural nature of his mathematics experiences:

> "It was something where you had to just remember in which order you did things, and that's it. It had no significance to me past that point at all."

Marcos' representation of *significance* is important – it speaks to the 'meaning' (Damon, 2008) that students may develop, or not develop, when learning school subjects. Marcos felt that the mathematics he worked on in school had no or little meaning, which negatively impacted his ability to engage with the subject.

Whereas the adults from Amber Hill spoke with regret about school mathematics, the adults from Phoenix Park spoke in uniformly positive ways when describing their mathematics classes. They reflected upon maths lessons using words such as "brilliant", "ideal" and "brave". As Susan, one of the Phoenix Park young women described:

> "So I think they had a very good approach to teaching in most subjects. And, as I said, I remember maths being particularly good. I remember the teacher being particularly good also. So I think the way they taught it was fantastic. I remember a lot of people enjoying maths." (Susan, Phoenix Park).

In describing their enjoyment of school mathematics many of the Phoenix Park adults referred to the open-ness of the approach:

> "I think it was definitely more creative. We were never too much said like "this is going to be on your exam, you need to memorize this". That's another thing that I've had a problem with the education system—just the whole regurgitation just for the exams… I don't know, they might have been prepping us, but it didn't feel like it." (Neil, Phoenix Park)

The adults who had attended Phoenix Park told me that the mathematics they had learned was useful in their jobs, and they seemed to have moved seamlessly from their mathematics classrooms into the mathematical demands of the workplace. This speaks to the improved social class standing of the young adults, as did the students' reports of their relationships with mathematics knowledge, an idea which will be considered in the next section.

Exploring the Interview Data – Part 2. Relationships with Knowledge.

The students at Amber Hill and Phoenix Park had both learned the same mathematics content and the teachers who had taught them had been equally qualified, but because the students engaged in different practices as they were learning mathematics they learned to engage with mathematics differently. In the Amber Hill classrooms the students rehearsed content through short questions, practising methods they had been shown by their teacher. They learned to engage in traditional classroom practices, such as using cues from questions to know which method to use, to help them achieve success in their exercise book questions (see also Boaler, 2002a). At Phoenix Park the students learned to ask questions, to inquire using mathematics and to draw conclusions using mathematical evidence.

Eight years later when asked about the usefulness of their mathematics education, the two sets of ex-students spoke very differently about mathematics. At Amber Hill the adults considered my questions and then talked about content labels. For example, when I asked Trevor, from Amber Hill, if mathematics had been useful to him in life, he said:

> "Yeah. I mean stuff like pi and trigonometry, stuff like that. That's never really been useful to me since. I mean I don't think I really remember it anymore."

Simon spoke similarly:

> "I think I've pretty much never used any of it, I think. It's been pretty much me almost teaching myself again the bits that I need to know. I think it needs to be pulled into real-life examples where I would be able to see why I'm calculating what I'm calculating rather than numbers relating to other numbers for no apparent reason."

Helen gave a similar perspective:

> "I don't know… I mean other things like trigonometry, I don't think you use it really in everyday life, do you? I suppose maybe if you're measuring things and…but you don't really use it."

The adults from Amber Hill talked about mathematics in ways that many adults talk about it – as numbers, percentages, and trigonometry. The adults from Phoenix Park were distinctive in not doing this; instead they talked about a subject with many dimensions that went beyond itemized content knowledge. When I asked Andrew, from Phoenix Park, whether maths had been useful, he said:

> "I suppose there was a lot of things I can relate back to maths in school. You know it's about having a sort of concept, isn't it, of space and numbers and how you can relate that back. And then, okay, if you've got an idea about something and how you would then use maths to work that out. I suppose maths is about problem-solving for me. It's about numbers, it's about problem solving, it's about being logical"

What seemed noteworthy about the descriptions from the two schools, is not that the adults from Amber said they did not use or had forgotten the content but that they only spoke about content. The Phoenix Park adults talked about mathematics in much broader ways. This reflected the 'multi-dimensional' mathematics (Boaler, 2008a) they had learned and the different forms of engagement they had been offered. In the following extract Stephen, who had made his way up to senior levels of hotel management, contrasted an approach to education that was just focused on content with his own education at Phoenix Park that had taught him to problem solve and "find ways around" difficulties in work.

> "I mean it's like anything. But especially with education, if you look at education on face value, what you're actually taught in school, to me, it just seems complete nonsense. It's the way you manage it, it's the way you apply yourself to it, and the techniques you've learned. You teach yourself, in doing it—that's what I've actually used in the years to come. It's not the actual nits and grits of whatever it is—whatever subject—to

be honest. Because I think if you struggle at something, you find ways around it, don't you. That's what I took with me. Maybe—maybe—I don't know—but maybe it's the style of teaching then that's given me that. If somebody stood up in a class of 30 and wrote on the blackboard for half an hour and then we did the exercises, maybe I wouldn't think like that, maybe I would just think it's about working out percentages." (Simon, Phoenix Park).

In the extract above Simon, himself, draws a distinction between, on the one hand, learning ways of being, which include "teaching" and "applying" yourself , engaging in "struggle" and "finding ways round" difficulties and on the other, learning methods such as 'working out percentages'.

When the students finished school, I argued that the Phoenix Park students were more successful in authentic mathematical situations because they had learned to engage in practices in school that were similar to those elsewhere, such as the 'real world'. Whereas the Amber Hill students had learned to follow textbook and teacher cues and to repeat a procedure, the Phoenix Park students learned to choose from different methods, adapt and apply methods, and draw from resources in their environment (Greeno, 1991) to help them work. I argued then that these opportunities had also given the students opportunities to develop different identities as learners (Boaler, 2002b). The interviews with the young adults added depth to this latter idea and extended it. The adults talked in ways that reflected a very different positioning to knowledge and life that involved working with responsibility and agency (Boaler & Greeno, 2000). Neil had worked his way up to a senior level in banking. After he had recalled his school mathematics experiences at Phoenix Park, he said:

"I mean I prefer to work in that way now, and that maybe comes from that. Like, you know, I'd much rather work and be given responsibility for doing a job, and not be…not have a sort of manager who's always watching what I do and trying to guide me all the time in terms of telling me exactly what to do all the time. I prefer being given responsibility to do something and doing it, and then presenting my results—which is similar to the way we learned in maths."

Sarah, who was learning to be a teacher, spoke in similar ways talking about the mathematical responsibility she had learned in determining the correctness of mathematical solutions:

"Um…well I mean I suppose these projects that we did, and the work that we did. If you could prove that the answer that you had, and the solution that you…the way that you used to work it out worked for you, and worked with a generic…you know, if you were able to work it out and prove that it works, that somebody else could also do it, but did it slightly differently. And they were both right. And it wasn't just a cross or a tick, you know?—which is what a lot of maths is. You know, a child faced with a page full of sums, and they're either right or wrong in a lot of cases—and that's not always the case in life."

Sarah talked about learning that different people may use different methods and solve problems in different ways but also come to the same conclusion. She also stated that what is important in knowing whether solutions are correct, is mathematical proof, rather than the words of a teacher or book. She contrasted the act of proving an answer with receiving a "cross or tick". This suggests a higher level of mathematical authority and responsibility, something that Neil also referred to when he spoke about presenting ideas in maths class as a child and now in his work in banking.

At Phoenix Park the teachers did not focus only upon mastery of content; their goals were much bigger. They wanted to develop inquiring, problem solving, responsible, young people. In the interviews with the young adults, it seemed that they had achieved this. The adults indicated that they had learned to take intellectual authority, as well as a very active stance towards mathematics in their lives. In the following extract Andrew spoke about his approach to examinations, contrasting the approach he learned to one taken by many other students (such as those at Amber Hill) who want to follow step-by-step procedures:

> "I tend to go into them anyway just thinking that I can do them. Yeah, there wasn't any time I thought, "Well, this is this, this is this." I just worked it out from the knowledge that I had."

The act of working "it out from the knowledge I had" reflects an active approach to mathematics that I had witnessed the students engaging in many times during school. They had been taught to apply methods they had learned to new and different situations in order to solve them. Such acts give students the opportunity to work on mathematics with agency (Pickering, 1995 Boaler & Greeno, 2000). In different studies (Boaler & Greeno, 2000, Boaler & Sengupta-Irving, *in press*) I have found agency to be something that is important for students when they work on mathematics, for their own sense of identity as active, thinking people. Working with agency is also an act that Pickering found to be fundamental to the work of mathematicians and scientists (Pickering, 1995). Agency and authority often develop hand in hand in mathematics classrooms as students learn to make choices in mathematics and to determine the correctness both of choices and solutions. The interviews with the young adults indicated that they had developed different levels of authority, an idea which emerged, in part, from one particular question I asked them. I asked all of the adults about work situations when they encountered some mathematics they could not do. At Amber Hill the adults talked about getting help from other people, which is certainly a worthwhile strategy. For example, Ian reflected:

> "Well if it's at work, then I'm free to approach someone more senior than myself and they'll talk me through it. If it's at home, if I can't work it out for myself, I would try and find out from someone like a friend, or my parents. My mum's fairly clever in maths."

This seems reasonable, as the Amber Hill students had learned in school that the authority for mathematical correctness lay with teachers and books. At Phoenix Park the adults again responded very differently when asked the same question:

JB: What happens if you encounter maths you can't do?

C: Keep on going until I do. Because I wouldn't…if something is…if I can't do it, then it annoys me. I need to see something through really until the end. So I need to understand how I've done it. If I don't immediately understand it, then I will keep on going at it until I do understand it.

In these and other interview reflections, the adults from Phoenix Park demonstrate a strong sense of agency and authority. Expressions such as "I will keep on going at it until I do understand it", "I tend to go into them thinking that I can do them", "I prefer being given responsibility and doing it", and "If you struggle at something you find ways around it" were absent in the descriptions from Amber Hill adults who simply described mathematics as a list of content that had not been useful to them. Further the Phoenix Park adults stated that they had developed this active, inquiring approach to mathematics in their lives and work from the practices in which they had engaged as school students. In the final part of this paper, I will explore further the different mathematical identities that the adults seemed to have developed, using lenses from both situated theory and from Giyoo Hatano's notion of 'adaptive expertise' (Hatano & Oura, 2003). Before doing this I will turn briefly to the students' reflections on the ways in which ability grouping in Amber Hill and mixed ability grouping in Phoenix Park had changed the opportunities that were available to them.

Exploring the Interview Data – Part 3. Ability Grouping.

The students at Amber Hill had experienced a version of ability grouping in school that is typical for students in England. At the age of 13 they were placed into 'sets' from set 1 to set 8, with set 1 students being prepared for the highest examination grades and students in lower sets being prepared for the lower grades. At the time of conducting my three-year study of the students, I learned that some students from set 2 downwards gave up on mathematics when they were put into a low set. There are, of course, advantages to providing the correct level of work to students so that they feel both challenged and supported, but the act of determining what level students are capable of working on for three years ahead and grouping them accordingly seemed to include significant disadvantages for the students' self-image and concurrent motivation to work hard in mathematics.

At Phoenix Park the teachers did something very unusual for a school in England: they kept students in mixed ability groups for almost three years, until a few weeks before the national examination when they placed them into one of three examination groups. My initial study produced a number of findings about the impact of ability grouping (see Boaler, a, b, c). In interviews ability grouping emerged as an important issue for the adults who had attended Amber Hill. Many of them spoke about the ways the ability grouping had shaped their whole experience of school.

Adults from Amber Hill who had been in set 2 downwards talked about the limits that had been put on their achievement. Marcos spoke passionately about this, saying:

> "You're putting this psychological prison around them (…), it's kind of… people don't know what they can do, or where the boundaries are, unless they're told at that kind of age."

Marcos' reminder that the placing of students into ability groups serves to give them powerful messages about their potential – messages that may serve as "psychological prisons" seems poignant. Later in the interview he expanded further saying:

> "It kind of just breaks all their ambition … particularly schools like *Amber Hill* where it's predominantly working-class kids whose parents don't necessarily have the ambition *for* them. And then if it's being reinforced in the classroom with kind of "yes you're going to be a labourer for the whole of your life" then it means they can't break out of that box. It's quite sad that there's kids there that could potentially be very, very smart and benefit us in so many ways, but it's just kind of broken down from a young age. So that's why I dislike the set system so much—because I think it almost formally labels kids as stupid." (Marcos, ex-Amber Hill student).

Mary, a bright and engaging young woman, spoke personally about her experiences of being in set 8 when they were given examination papers to work on:

> "I remember now…I done the next paper up to my set, but I don't know what my mark was actually based on—probably the set 8 one. But I think they said, "You can do the next paper if you want to." And I did. But I think I always felt like I needed… because it was so late in doing the GCSEs, I think some of it started to sort of click into place with this teacher. I thought, "Oh, if only I could go up a set." But it was a bit too late for that."

When I asked Mary how her mathematics experience in school could have been better she talked about her discomfort in asking her male teachers any questions and said she would have liked:

> "Someone who I felt comfortable with, and someone who encouraged me to ask and to question, and made me feel comfortable asking. And definitely not just learning formulas and that, but relating them to real life, being given situations where you would use certain things and *how* you would use them. I think yeah."

Mary, like many of the Amber Hill adults, regretted the procedural nature of her mathematics experience as well as the lack of opportunity she had to ask questions and to feel comfortable in doing that. But it was not only Amber Hill adults who had been in low sets who communicated the problems of ability grouping; even those in the top set talked about having to wait for students to catch up or having to work too quickly to keep up with others. The model of ability grouping that is used in England (and other countries) is based upon the idea that students can be divided into groups according to their capability and taught content in a targeted way at a particular pace. This fixed pacing had caused problems for students even after dividing them into such narrow bands:

> "So you'd be in 2nd set, and you were kind of learning, and you were kind of encouraged to try and get to the 1st set; but then almost once you're there, things are going too fast for you to actually know what the hell is going on." (Marcos, Amber Hill)

In the Phoenix Park adults' reflections on school they often cited the opportunity they received to work at their own pace and take ideas to different levels as part of the reason for their enjoyment of mathematics. Darren contrasted the freedom of mathematics to science, the one subject at Phoenix Park that was taught traditionally using ability groups:

> "I know from science lessons and stuff, people just standing there… I mean it depends on the teachers, but someone just standing there rattling off a load of stuff, and you write it down, it tends to go in one ear and out the other—or you just tend to get bored, so you lose interest. Whereas I know in maths, it may have been my own thing that I did quite like maths, but I was always interested in it. And I felt like I could go as fast as I wanted. And you didn't feel like you had to wait for people, or that you were trying to catch up to people."

Angus, a high achieving student who earned the highest examination grade at Phoenix Park, also reflected on the freedom to take work to appropriate levels:

> "I suppose again everyone was working at different levels. And some people would take some things further than others. But I don't remember having to—you know, if you particularly excel in a subject—having to wait for other people to catch up." (Angus, Phoenix Park).

Angus spoke in strong contrast to the Amber Hill students, about *not having to wait* for others – even though he learned in mixed achievement groups. The main rationale for ability grouping is the provision of work that is appropriate in level for different students, yet it was the Amber Hill students, who were placed into finely divided groups, who complained that work was too hard, easy, fast or slow. At Phoenix Park the students often started work on the same or similar tasks but were then encouraged to take them in different directions and to different levels. This served to differentiate the work (Tomlinson, 1999) and for the students to feel that the work was at the right level for them. This form of grouping and differentiation took considerable teacher expertise.

Teachers in different countries have very different ideas about the ideas that should be communicated to students about their potential. It is noticeable that teachers from Japan, for example, speak in these ways about students: "In Japan what is important is balance. Everyone can do everything; we think that is a good thing … so we can't divide by ability". (Boaler, 2008b, p. 108). Japanese teachers seem to regard their job as providing work that is appropriate for everyone and holding high expectations for all students. In England and other countries that have a long history of ability grouping there are deeply held, cultural beliefs about the ways students should be grouped and encouraged (Altendorff, 2012). Many teachers from England simply regard the labelling of children as "low ability" or a "level 2 child" as normal and find the idea of grouping children heterogeneously inconceivable (Boaler, Wiliam & Brown, 2000). Some of the students from Phoenix Park described their school approach as “brave”, and this seems to be an accurate description as the school went against the norms of the country in grouping children heterogeneously and teaching them through open projects. Phoenix Park school no longer works in these ways, having succumbed to many years of conservative education policies (even before the Conservative party regained power in government) and the expectation that all schools group students by ability and teach in the same way (Boaler, 2009). But the data from the students who attended Phoenix Park, both in terms of their school achievement and their knowledge relationships and professional standing as adults, indicate that there is an important need to consider alternative ways of teaching students mathematics, even when these go against long-held traditions and cultural norms.

DISCUSSION AND CONCLUSION.

The interviews from this study represent a small sample, but the extreme differences communicated by the two sets of young adults as they talked about mathematics and about knowledge use make them interesting to consider. One analytical frame for making sense of the different responses is that of situated theory (Lave, 1988, Wenger, 1998). At Phoenix Park the students had learned to engage in practices that were very different from those at Amber Hill. These involved the students in acting with agency and authority – asking questions, choosing mathematical directions and determining the correctness of their work. By contrast the Amber Hill students who had learned mathematics content and performed very well in classroom exercises, engaged with mathematics passively, learning to practice content by rehearsing methods and checking for correctness with the teacher or the book. These ways of engaging seemed to have had an important impact on the young people as adults, leading them to interact with knowledge and work tasks differently.

Greeno (2006) considers the work that students are authorized and held accountable to do in classrooms and reflects upon a case in which students were held accountable for constructing representations. At Phoenix Park the students were authorized to act with agency and responsibility and they were held accountable for mathematical reasoning and sense making. The participation structures in which they learned to be successful at Phoenix Park included choosing methods to help make sense of problems, using and applying methods and determining the correctness of their solutions. As students learned to become more effective in these participation structures they learned to engage with mathematics in a particular way and to develop relationships with knowledge that were very different from the Amber Hill adults. The adults who had attended Phoenix Park talked with confidence about tackling any problem they encountered and saw mathematics as knowledge that they could use. They were prepared to “keep on going at it”, “struggle”, “find ways round” problems and take “responsibility”. Their words seem to reflect both actions and beliefs that combine in the development of more active and capable mathematical identities than those held by the Amber Hill adults. At Amber Hill the adults seemed to relate to mathematics – or at least school mathematics – very differently, communicating a less active stance towards the subject that did not include working with agency and authority. Instead they had dismissed most of school mathematics as an irrelevance to their work and lives.

Situated theories have important implications for education, because they lead researchers to consider the ways students engage with mathematics as well as the knowledge they hold in their heads. The two sets of adults I interviewed had scored at the same levels on national examinations but it was clear that they had learned to relate to mathematics knowledge very differently. A cognitive researcher may conclude that the Phoenix Park adults understood mathematics in more depth as they had developed conceptual understanding, and therefore they could use mathematics more. This has some validity but it seems inadequate in capturing the differences between the adults, in particular the active, and confident ways in which the Phoenix Park adults used mathematics in their lives.

Another analytical frame for making sense of the differences between the two sets of adults comes from Giyoo Hatano and his colleagues, who developed the idea of 'adaptive expertise'. Hatano & Oura (2003) proposed that those with adaptive expertise are characterized by their 'innovative and creative competencies' (p28). By contrast, those with 'routine expertise' are those who learn to solve familiar problems quickly. People with adaptive expertise also modify procedures, apply their ideas and respond flexibly to different situations. These two forms of expertise seem to reflect the capabilities of the Phoenix Park and Amber Hill students. Hatano himself read about the experiences of the Phoenix Park students when they were in school and concluded that they "were on the trajectory toward adaptive expertise" (Hatano & Oura, 2003, p28). That trajectory that was initiated by their mathematics teachers in school seems to have continued into adulthood and impacted their lives in quite profound ways.

The idea of adaptive expertise draws mainly from a cognitive lens, whereas the idea that the adults' developed different mathematical identities that included self-beliefs, ideas about mathematics, and an eagerness to engage actively with mathematics draws from a situated perspective. There has been some history of debates between those in the cognitive and situated camps (Anderson, Reder & Simon, 1997; Greeno, 1997) each arguing the importance of both. Greeno (1997) usefully pointed out that the situated perspective does not preclude cognitive understandings; nor does it dismiss the cognitive perspective. Instead it adds another dimension, pointing out that a person's competence in any domain draws in part from the knowledge they hold in their minds but also from the practices in which they are able to engage, practices that may be distributed between people and systems of an environment (Pea, 1999). But, although situated theories may include cognitive understandings, they downplay them whereas cognitive theories focus squarely upon student conceptions. Hatano's ideas of adaptive expertise benefit from being broad; diSessa describes them as wonderfully 'balancing cognitive and social perspectives' (2007, p40). They also serve the purpose of highlighting not only the ways the Phoenix Park adults engaged with mathematics and life, but what they could do and the particular expertise they had developed. Martin and Schwartz (2007) point to the importance of adaptive expertise in twenty-first century work and remind us of the value of being able to adapt in changing circumstances. They also caution that there have been too few studies of the development of adaptive expertise or understanding of 'the characteristics of adaptive behaviors or the conditions that lead to them' (Martin & Schwartz, 2007 px). The adults of Phoenix Park, as well as their school experiences, may be an interesting resource in helping us understand the ways we may best prepare students for twenty-first century work (see also Boaler, 2008b).

The lenses of adaptive expertise and situated theory highlight different affordances of the problem solving approach of Phoenix Park, but both may be useful in helping policy makers understand the importance of high quality mathematics teaching approaches. The adaptive expertise lens is important in showing the forms of knowledge that students need in preparing for work and future study, and policy makers are often more attentive to cognitive analyses. But the situated lens is equally important in drawing attention to forms of engagement and to the limitations of closed tests in capturing the impact of different mathematics teaching approaches upon student capability and knowing in non-test situations such as life.

In the United States a new curriculum, the Common Core Standards for Mathematics, has been adopted in 48 of 53 states and will be implemented in the next few years. Importantly, this new curriculum includes standards for ‘mathematical practices’, such as reasoning, sense making and proving. These practices have been absent from mathematics classrooms in the United States for many years, but their enshrinement in the new curriculum, and hopefully the assessments that will accompany them, mark an opportunity to shift mathematics classrooms further towards the problem solving environments of Phoenix Park. This shift will require significant changes in teachers’ practice and considerable demands for teacher learning. The forms of teacher learning opportunities that are provided to teachers in making this shift may turn out to be critical to the success of the new curriculum (Ball & Cohen, 1999; Stein &Brown, 1997).

Opportunities to provide learning situations in which students interact with systems of their environment and engage in a broad range of mathematical practices are rare in mathematics classrooms. But the reflections of the Phoenix Park and Amber Hill adults suggest that such experiences may not only enhance individual understanding but also provide students with opportunities to develop adaptive expertise and to engage actively with mathematics in their lives. At Amber Hill the students had learned formal mathematical methods, but they had learned them through passive participation structures, and in later years they dismissed the methods as irrelevant. At Phoenix Park the students engaged in mathematical reasoning and sense making, and they learned to act with agency and authority. These contrasting practices seemed to have played an important role in the development of different mathematical identities that for the Phoenix Park adults included intellectual freedom. For the Amber Hill adults intellectual freedom was replaced by ‘psychological imprisonment’ as Marcos so notably described it. In talking with the Amber Hill adults, it seemed that their mathematical identities included submission to outside authorities – canons of knowledge and lists of content. By contrast Phoenix Park adults talked about actively using knowledge to solve problems. They saw mathematics as knowledge that they could use and they were prepared to “struggle”, “find ways around” problems and take “responsibility”. It seems very likely that it was these differences that had enabled the adults to move up the social scale and gain more success than the Amber Hill adults, even those with comparable examination results, in their work and life.

Providing opportunities for students to develop active mathematical identities that include self-belief as well as adaptive expertise should be the goal of mathematics education across the world. If it is not, we run the risk of producing school students who are ever more disenchanted by the irrelevance of their school education. Instead of empowering students to use and apply their knowledge in ever changing circumstances we may forever be reminding them of the mismatch between the mathematics they learned in school and the mathematics they need for today's innovative, adaptive and technological workplace. At Phoenix Park the teachers achieved something remarkable – they encouraged all students to high levels, enabling many of them to achieve higher examination grades than would have been expected. At the same time they gave students an experience of mathematics that led them to enjoy the subject, to appreciate its importance and to develop active, capable relationships with knowledge that extended well beyond their mathematics classrooms.

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